Equality & Equivalence

Year 7

#MathsEveryoneCan

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Autumn 1: Algebraic thinking

Week 1: Exploring Sequences

Rather than rushing to find rules for n^{th} term, this week is spent exploring sequences in detail, using both diagrams and lists of numbers. Technology is used to produce graphs so students can appreciate and use the words "linear" and "non-linear" linking to the patterns they have spotted. Calculators are used throughout so number skills are not a barrier to finding the changes between terms or subsequent terms. Sequences are treated more formally later this unit.
National curriculum content covered:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- make and test conjectures about patterns and relationships
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- generate terms of a sequence from a term-to-term rule
- recognise arithmetic sequences
- recognise geometric sequences and appreciate other sequences that arise

Weeks 2 to 4: Understanding and using algebraic notation

The focus of these three weeks is developing a deep understanding of the basic algebraic forms, with more complex expressions being dealt with later. Function machines are used alongside bar models and letter notation, with time invested in single function machines and the links to inverse operations before moving on to series of two machines and substitution into short abstract expressions.
National curriculum content covered:

- move freely between different numerical, algebraic, graphical and diagrammatic representations
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- recognise and use relationships between operations including inverse operations
- model situations or procedures by translating them into algebraic expressions
- substitute values in expressions, rearrange and simplify expressions
- use and interpret algebraic notation, including:
  - ab in place of $a \times b$
  - $3y$ in place of $y + y + y$ and $3 \times y$
  - a^2 in place of $a \times a$
  - ab in place of $a \times b$
  - $\frac{a}{b}$ in place of $a \div b$
- generate terms of a sequence from a term-to-term rule
- produce graphs of linear functions of one variable

Weeks 5 and 6: Equality and equivalence

In this section students are introduced to forming and solving one-step linear equations, building on their study of inverse operations. The equations met will mainly require the use of a calculator, both to develop their skills and to ensure understanding of how to solve equations rather than spotting solutions. This work will be developed when two-step equations are met in the next place value unit and throughout the course. The unit finishes within consideration of equivalence and the difference between this and equality, illustrated through collecting like terms.
National curriculum content covered:

- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- simplify and manipulate algebraic expressions to maintain equivalence by collecting like terms
- use approximation through rounding to estimate answers
- use algebraic methods to solve linear equations in one variable

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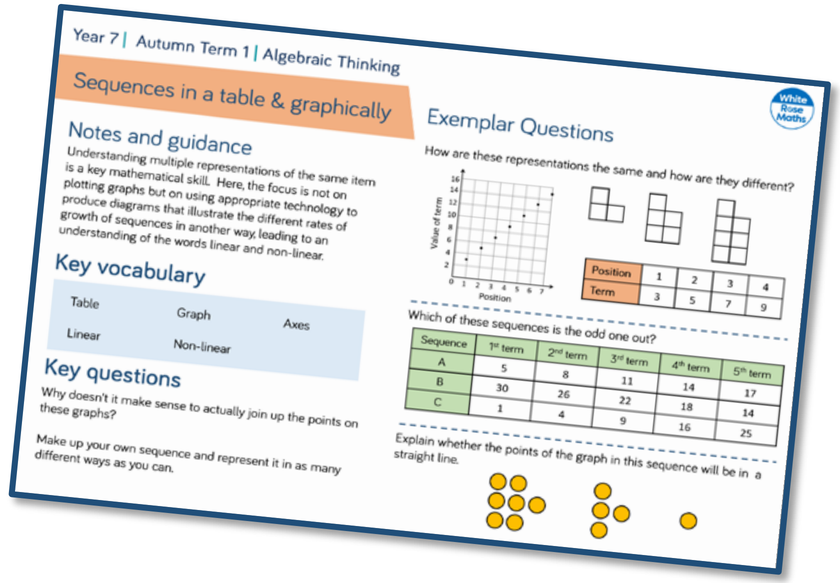
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

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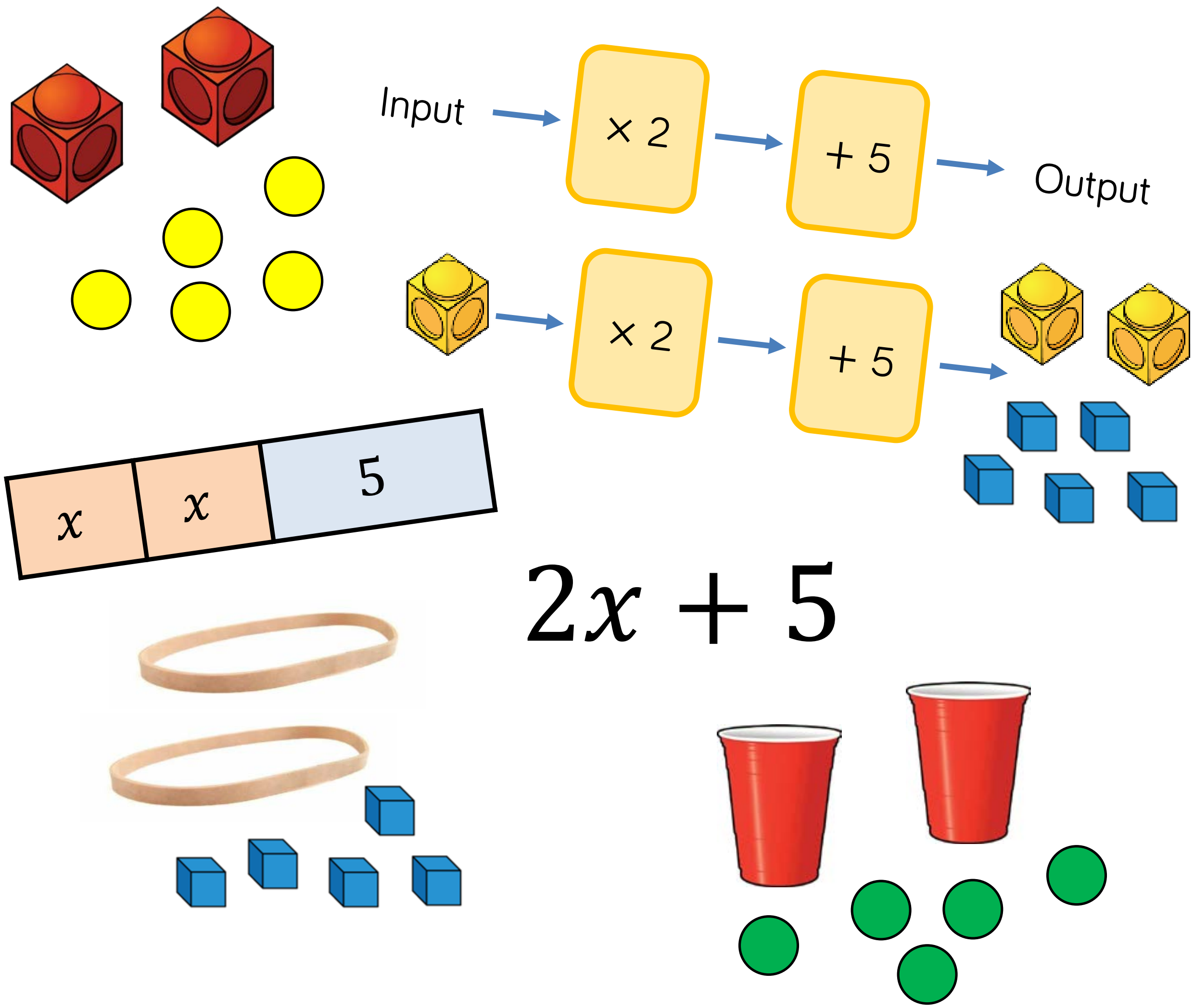
Key Representations



Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas for how you might represent algebra. Cups, cubes and elastic bands lend themselves well to representing an unknown, whereas ones (from Base 10) and counters work well to represent a known number.

Be careful to ensure that when representing an unknown students use equipment that does not have an assigned value – such as a Base 10 equipment and dice.

Equality and Equivalence

Small Steps

- Understanding the meaning of equality
- Understand and use fact families, numerically and algebraically
- Solve one-step linear equations involving $+/-$ using inverse operations
- Solve one-step linear equations involving $\times/\div$ using inverse operations
- Understand the meaning of like and unlike terms
- Understanding the meaning of equivalence
- Simplify algebraic expressions by collecting like terms, using the $\equiv$ symbol

Understanding equality

Notes and guidance

Students often misinterpret the equals sign as "makes". The bidirectional nature of equality needs to be emphasised so that students realise the left hand side and right hand side of an equation are worth the same amount rather than making each other. It is helpful to read the equals sign as "is equal to" to support this.

Key vocabulary

Equality	Equation	Equals
Is equal to		

Key questions

What difference does it make when you swap the right hand side and the left hand side of an equation?

If you change the order of the terms on one side of an equation, will it still be true?

Exemplar Questions

Which of the following are true?

- $6 + 3 = 9$
- $12 + 9 = 3 \times 7$
- $8 = 5 + 3$
- $8 \div 0.2 = 80 \div 2$
- $5 + 6 = 8 + 3$
- $6700 - 67 = 99 \times 67$
- $312 + 99 = 312 + 100 - 1$

Here is a number wall.

<table>
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<tr><td colspan="7">7</td><td colspan="1">1</td><td colspan="4">4</td></tr>
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How many equations can you find from this number wall?
E.g. $12 = 3 + 9, \; 4 \times 3 = 7 + 1 + 4$

Work out the missing numbers in these equations.

$7 + 8 = 10 + ?$

$9 + ? = 20 - 6$

$12 \times 5 = ? + 50$

$? \div 2 = 60 \div 4$

Understand and use fact families

Notes and guidance

Students will be familiar with fact families from their work in previous key stages. This small step extends their knowledge to algebraic fact families in preparation for solving equations and recognising equivalent forms of the same equation.

Key vocabulary

Fact family	Bar Model	Is equal to

Key questions

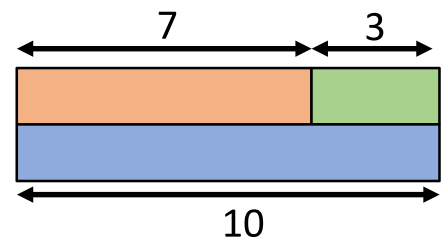
Do bar models need to be drawn to scale?

If you know one addition fact, how many subtraction facts do you also know?

Exemplar Questions

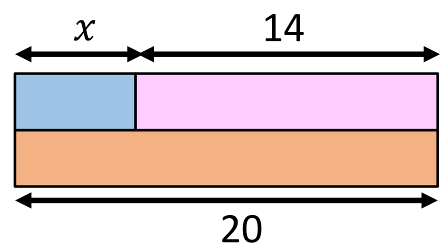
This bar model shows::

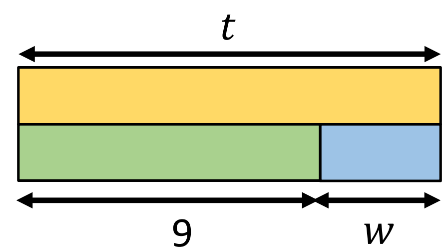
$$7 + 3 = 10$$
$$7 = 10 - 3$$



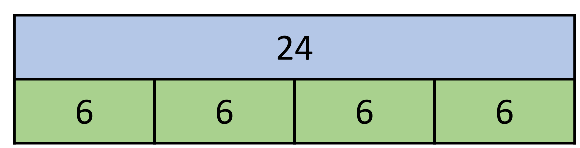
What other facts does it show?

Write the fact families for these bar models.





Write the fact family for this bar model.



$$67.3 - 11.6 = 55.7$$

Using the fact above, write down an addition and another subtraction.

Solve one-step equations (+/−)

Notes and guidance

Calculators should be used to find the solutions to the one-step equations in this step. Avoid equations such as $x + 3 = 8$ as students are likely to "spot" the answer rather than use the inverse operation. Practice questions should include the unknown on either sides and terms in any order. Link also to function machines.

Key vocabulary

Equation	Solve	Solution
Unknown	Inverse	

Key questions

What's the difference between an equation and an expression?

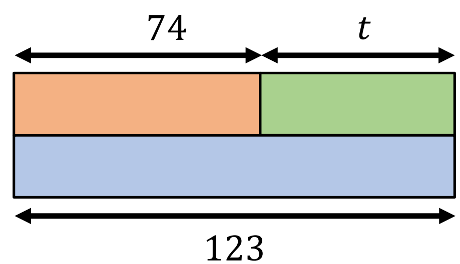
How is an 'unknown' different from a 'variable'?

What is the inverse of 'add on 12'?

Exemplar Questions

Draw a bar model to illustrate $x + 37 = 61$
Write down the fact family for your bar model.
What is the value of x?

What additions does this diagram show?
What subtractions does it show?



What is the value of t?

Solve these equations.

$a + 47 = 93$	$38 = 5.6 + b$	$36 = c - 102$
$4.8 + d = 11$	$91 = 53 + e$	$70 - f = 11.4$

Ken thinks of a number. He subtracts 78 from his number and gets the answer 137. Show this information as an equation and solve the equation to find Ken's number.
How else could you represent the information?

Solve one-step equations (×/÷)

Notes and guidance

Calculators should again be used to find the solutions to these equations involving multiplication and division. This will allow for a variety of questions, avoiding misconceptions such as answers always been integers. Emphasis should again be on inverse operations with unknowns in different places, not just $ax = b$.

Key vocabulary

Equation	Solve	Solution
Unknown	Inverse	

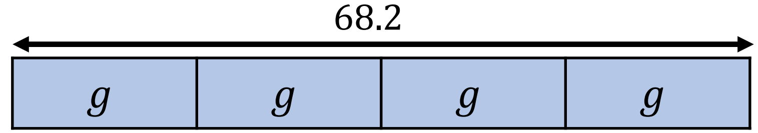
Key questions

Are the equations $3x = 192$ and $192 = 3y$ the same or different?

How can we check the answers to our equations are correct?

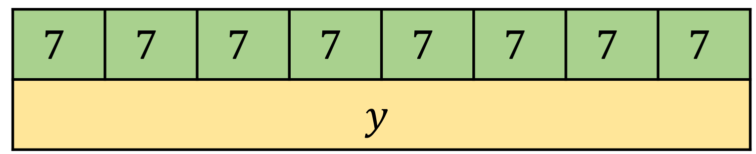
Exemplar Questions

Write the fact family for this bar model.



Work out the value of g.

Write the fact family for this bar model.



Work out the value of y.

Marta thinks of a number.

She divides her number by 7 and gets the answer 42

Write this information as an equation.

Solve your equation to find Marta's number.

Solve these equations.

$\frac{a}{23} = 9$ $\quad 46 = \frac{b}{11}$ $\quad 187 = 5c$ $\quad 12d = 3$

Understand like and unlike terms

Notes and guidance

This small step is vital in supporting simplification of algebraic expressions. Displaying a pair of terms on the board and using a true/false activity with mini-whiteboards is a good way to assess.

Key vocabulary

Term	Like	Unlike
Coefficient	Index	

Key questions

Why are $3x$ and $3x^2$ unlike terms?

What is the coefficient of d in the term $-7d$?

Exemplar Questions

Like terms	Unlike terms
$5a, 6a$	$5a, 5b$
$10t, -3t$	$-10t, -3$
$2xy, 4xy$	$2xy, 4xz$
$10. -7$	$10, 7a$
$3a^2, 7a^2$	$3a^2, 7b^2$

Explain why the terms in the left are called 'like terms' and the terms on the right are called 'unlike terms'.

Sort the expressions below into sets of like terms

3	$3a$	$-3b^2$	$-3a$
a^2	-3	$6a$	-6
$6b^2$	6	$-6a$	$12b$

Jemima says $3xy$ and $6yx$ are like terms because the expressions involve the same powers of the same letters just in a different order. Do you agree?

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The meaning of equivalence

Notes and guidance

Students often get confused between equality and equivalence and try to 'solve' when they are asked to simplify. This step is needed to illustrate the difference and to start to appreciate the idea of an identity as opposed to an expression or an equation. It is worth noting that repeated substitution is a demonstration rather than a proof.

Key vocabulary

Expression	Term	Expression
Equivalent		

Key questions

Are the expressions $2x$ and x^2 equivalent?
Why or why not?

Write down as many expressions as you can that are equivalent to $5p$

Exemplar Questions

Substitute $x = 7$ into each of these expressions.

$5x$	$2x$	$8x - 3x$	$x + x$
$2 + 4x$	$3x + 2x$	$6x - x$	$4x + 2$

Which expressions give you the same answers? Why?
Repeat with a different value of x.
What do you notice?

Which of the following expressions are equivalent?

$6m$	$2m + 4m$	$\frac{m}{6}$	$10m - 4m$
$2m \times 3$	$m + 6$	$30m \div 5$	$4m - m + 3m$

Check by substituting several values of m.

Work out the expressions below for several values of y.

$2y + 10$ $2(y + 5)$

What do you notice. Will this always be the case?

Collect like terms using ≡ symbol

Notes and guidance

Building on the last two small steps, we now move to simplifying expressions by collecting like terms. It is important to write e.g. $3x + 2x \equiv 5x$ rather than the commonly seen $3x + 2x = 5x$ and to discuss with the students that the former is true for any value of x, whereas equations are only true for specific values of x.

Key vocabulary

Like/unlike	Equation	Equivalent
≡	Simplify	Collect

Key questions

What's the difference between equality and equivalence?

Can you simplify unlike terms?

Exemplar Questions

Which of the following are true and which are false?

$6x + 2x \equiv 8x$ $\quad$ $6x - 2x \equiv 4x$ $\quad$ $2x \equiv 8x \div 4$

$3x + 2 \equiv 5x$ $\quad$ $3x + 2y \equiv 5xy$ $\quad$ $x + 2 \equiv 2 + x$

$5x - 5 \equiv x$ $\quad$ $10x \equiv 5x \times 2$

Simplify these expressions so they have only one term.

$7a + 2a$	$3a + 4a + 5a$	$10b - 3b + 5b$
$6x^2 + 5x^2$	$2ab + 6ab - 3ab$	$10 + 6 - 3$

Correct the mistakes in the simplifications below.

$5x + 3x \equiv 8x^2$ $\quad$ $10y - 3y \equiv 13y$ $\quad$ $9p + 4p = 94p$

Simplify the expressions below by collecting like terms

$3a + 4 + 5a$ $\quad$ $6b + 2c - 2b + 6c$ $\quad$ $5d + 3e + 2d - 3e$

Find expressions that simplify to $8x + 10y$