Autumn Scheme of Learning

Year 5/6

#MathsEveryoneCan

2019-20
How to use the mixed-age SOL

In this document, you will find suggestions of how you may structure a progression in learning for a mixed-age class.

Firstly, we have created a yearly overview.

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</table>

For each block of learning, we have grouped the small steps into themes that have similar content. Within these themes, we list the corresponding small steps from one or both year groups. Teachers can then use the single-age schemes to access the guidance on each small step listed within each theme.

The themes are organised into common content (above the line) and year specific content (below the line). Moving from left to right, the arrows on the line suggest the order to teach the themes.

Each term has 12 weeks of learning. We are aware that some terms are longer and shorter than others, so teachers may adapt the overview to fit their term dates. The overview shows how the content has been matched up over the year to support teachers in teaching similar concepts to both year groups. Where this is not possible, it is clearly indicated on the overview with 2 separate blocks.
How to use the mixed-age SOL

Here is an example of one of the themes from the Year 1/2 mixed-age guidance.

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<tr>
<td>- Subtract with 2-digits (2)</td>
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<td>- Find change - money</td>
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</table>

In order to create a more coherent journey for mixed-age classes, we have re-ordered some of the single-age steps and combined some blocks of learning e.g. Money is covered within Addition and Subtraction.

The bullet points are the names of the small steps from the single-age SOL. We have referenced where the steps are from at the top of each theme e.g. Aut B2 means Autumn term, Block 2. Teachers will need to access both of the single-age SOLs from our website together with this mixed-age guidance in order to plan their learning.

Points to consider
- Use the mixed-age schemes to see where similar skills from both year groups can be taught together. Learning can then be differentiated through the questions on the single-age small steps so both year groups are focusing on their year group content.
- When there is year group specific content, consider teaching in split inputs to classes. This will depend on support in class and may need to be done through focus groups.
- On each of the block overview pages, we have described the key learning in each block and have given suggestions as to how the themes could be approached for each year group.
- We are fully aware that every class is different and the logistics of mixed-age classes can be tricky. We hope that our mixed-age SOL can help teachers to start to draw learning together.
In this section, content from single-age blocks are matched together to show teachers where there are clear links across the year groups. Teachers may decide to teach the lower year’s content to the whole class before moving the higher year on to their age-related expectations. The lower year group is not expected to cover the higher year group’s content as they should focus on their own age-related expectations.

In this section, content that is discrete to one year group is outlined. Teachers may need to consider a split input with lessons or working with children in focus groups to ensure they have full coverage of their year’s curriculum. Guidance is given on each page to support the planning of each block.

The themes should be taught in order from left to right.
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<td>Y6: Number: Algebra</td>
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<tr>
<td>Geometry: Properties of Shape</td>
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</tbody>
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In this block, children build on their previous knowledge of what a fraction is. Year 5 look at using multiplication and division to find equivalent fractions whilst Year 6 apply these skills to start to simplify fractions.

Both year groups add and subtract fractions with the same denominator and denominators that are multiples of the same number whilst Year 6 move on to adding and subtracting fractions where the denominators are not multiples of the same number.
Fractions (2)

Common Content

**Multiplication**
- Year 5 (Spr B2)
  - Multiply unit fractions by an integer
  - Multiply non-unit fractions by an integer
  - Multiply mixed numbers by an integer
- Year 6 (Aut B3)
  - Multiply fractions by integers
  - Multiply fractions by fractions

**Fraction of an amount**
- Year 5 (Spr B2)
  - Fraction of an amount
  - Using fractions as operators
- Year 6 (Aut B3)
  - Fraction of an amount
  - Fraction of an amount - find the whole

**Division**
- Year 6 (Aut B3)
  - Divide fractions by integers (1)
  - Divide fractions by integers (2)

**Four operations**
- Year 6 (Aut B3)
  - Four rules with fractions

Both year groups multiply fractions by integers, with Year 6 then moving on to multiply a fraction by a fraction.

Year 6 also explore dividing fractions and draw their learning together by using any of the four operations with fractions.

Both year groups find fractions of an amount, using bar models to support their understanding.
Block 3 – Fractions

Theme 1 – Equivalence and simplifying
Equivalent Fractions

Notes and Guidance

Children explore equivalent fractions using models and concrete representations.

They use models to make the link to multiplication and division. Children then apply the abstract method to find equivalent fractions.

It is important children have the conceptual understanding before moving on to just using an abstract method.

Mathematical Talk

What equivalent fractions can we find by folding the paper? How can we record these?

What is the same and what is different about the numerators and denominators in the equivalent fractions?

How does multiplication and division help us find equivalent fractions? Where can we see this in our model?

Varied Fluency

Take two pieces of paper the same size. Fold one piece into two equal pieces. Fold the other into eight equal pieces. What equivalent fractions can you find?

Use the models to write equivalent fractions.

Eva uses the models and her multiplication and division skills to find equivalent fractions.

Use this method to find equivalent fractions to $\frac{2}{4}$ and $\frac{4}{4}$ where the denominator is 16.

Eva uses the same approach to find equivalent fractions for these fractions. How will her method change?

$\frac{4}{12} = \boxed{\phantom{0}}$  $\frac{6}{12} = \boxed{\phantom{0}}$  $\frac{6}{12} = \boxed{\phantom{0}}$
Equivalent Fractions

Reasoning and Problem Solving

Rosie says,

To find equivalent fractions, whatever you do to the numerator, you do to the denominator.

Using her method, here are the equivalent fractions Rosie has found for $\frac{4}{8}$:

\[
\frac{4}{8} = \frac{8}{16} \quad \frac{4}{8} = \frac{6}{10}
\]

\[
\frac{4}{8} = \frac{2}{4} \quad \frac{4}{8} = \frac{1}{5}
\]

Are all Rosie’s fractions equivalent? Does Rosie’s method work? Explain your reasons.

Ron thinks you can only simplify even numbered fractions because you keep on halving the numerator and denominator until you get an odd number.

Do you agree? Explain your answer.

Here are some fraction cards. All of the fractions are equivalent.

\[
\frac{4}{A} \quad \frac{B}{C} \quad \frac{20}{50}
\]

$A + B = 16$

Calculate the value of $C$.

Ron is wrong. For example $\frac{3}{9}$ can be simplified to $\frac{1}{3}$ and these are all odd numbers.

$A = 10$
$B = 6$
$C = 15$
Simplify Fractions

Notes and Guidance

Children use their understanding of the highest common factor to simplify fractions, building on their knowledge of equivalent fractions in earlier years. Children apply their understanding when calculating with fractions and simplifying their answers. Encourage children to use pictorial representations to support simplifying e.g. a fraction wall.

Mathematical Talk

Can you make a list of the factors for each number? Which numbers appear in both lists? What do we call these (common factors)? What is the highest common factor of the numerator and denominator? Is a simplified fraction always equivalent to the original fraction? Why? If the HCF of the numerator and denominator is 1, can it be simplified?

Varied Fluency

Alex is simplifying $\frac{8}{12}$ by dividing the numerator and denominator by their highest common factor.

Factors of 8: 1, 2, 4, 8
Factors of 12: 1, 2, 3, 4, 6, 12
4 is the highest common factor.

Use Alex's method to simplify these fractions:

\[
\frac{6}{9}, \frac{6}{18}, \frac{10}{18}, \frac{10}{15}, \frac{15}{50}
\]

Mo has 3 boxes of chocolates. 2 boxes are full and one box is $\frac{4}{10}$ full.

To simplify $\frac{4}{10}$ keep the whole number the same and simplify the fraction. $\frac{4}{10}$ simplifies to $\frac{2}{5}$

Use Mo's method to simplify:

\[
3 \frac{4}{8}, 5 \frac{9}{21}, 2 \frac{7}{21}, \frac{32}{10}, \frac{32}{6}
\]
Simplify Fractions

Reasoning and Problem Solving

Find the total of the fractions. Give your answer in its simplest form.

\[
\frac{5}{9} + \frac{1}{9} = \frac{5}{9} + \frac{3}{9} = \frac{5}{9} + \frac{7}{9} = \]

Do all the answers need simplifying? Explain why.

\[
\frac{5}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3} \\
\frac{5}{9} + \frac{3}{9} = \frac{8}{9} \\
\frac{5}{9} + \frac{7}{9} = 1\frac{3}{9} = 1\frac{1}{3} \\
\frac{8}{9} \text{ does not need simplifying because the HCF of 8 and 9 is 1}
\]

Tommy is simplifying \( \frac{4}{15} \)

\[
\frac{4}{15} = 1\frac{3}{4}
\]

Explain Tommy’s mistake.

Tommy has divided the whole number by 4 instead of just simplifying \( \frac{12}{16} \) by dividing the numerator and denominator by 4.

Sort the fractions into the table.

<table>
<thead>
<tr>
<th>Simplifies to ( \frac{1}{2} )</th>
<th>Simplifies to ( \frac{1}{3} )</th>
<th>Simplifies to ( \frac{1}{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{15} )</td>
<td>( \frac{2}{16} )</td>
<td>( \frac{8}{16} )</td>
</tr>
<tr>
<td>( \frac{4}{16} )</td>
<td>( \frac{5}{10} )</td>
<td>( \frac{3}{9} )</td>
</tr>
<tr>
<td>( \frac{6}{12} )</td>
<td>( \frac{2}{8} )</td>
<td></td>
</tr>
</tbody>
</table>

Simplifies to \( \frac{1}{2} \):

\( \frac{2}{4} \), \( \frac{8}{16} \), \( \frac{5}{10} \), \( \frac{6}{12} \)

Simplifies to \( \frac{1}{3} \):

\( \frac{5}{15} \), \( \frac{3}{9} \), \( \frac{10}{15} \)

Simplifies to \( \frac{1}{4} \):

\( \frac{4}{16} \), \( \frac{2}{8} \)

Can you see any patterns between the numbers in each column?

What is the relationship between the numerators and denominators?

Can you add three more fractions to each column?

Complete the sentence to describe the patterns:
When a fraction is equivalent to \( \frac{1}{2} \), the numerator is ________ the denominator.

When a fraction is equivalent to \( \frac{1}{3} \), the numerator is ________ the denominator.

Repeat for \( \frac{1}{4} \) and \( \frac{1}{4} \).
Fractions on a Number Line

Notes and Guidance

Children count forwards and backwards in fractions. They compare and order fractions with the same denominator or denominators that are multiples of the same number. Encourage children to draw extra intervals on the number lines to support them to place the fractions more accurately. Children use the divisions on the number line to support them in finding the difference between fractions.

Mathematical Talk

Which numbers do I say when I count in eighths and when I count in quarters?

Can you estimate where the fractions will be on the number line?

Can you divide the number line into more intervals to place the fractions more accurately?

How can you find the difference between the fractions?

Varied Fluency

Jack is counting in quarters. He writes each number he says on a number line. Complete Jack’s number line.

Can you simplify any of the fractions on the number line?
Can you count forward in eighths? How would the number line change?

Place \( \frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{5}{8}, \frac{7}{8} \) and \( \frac{3}{16} \) on the number line.

Which fractions were the easiest to place?
Which fractions were the hardest to place?
Which fraction is the largest? Which fraction is the smallest?
What is the difference between the largest and smallest fraction?
Rosie is counting backwards in fifths. She starts at \(3 \frac{2}{5}\) and counts back nine fifths. What number does Rosie end on? Show this on a number line.

Rosie ends on \(1 \frac{3}{5}\)

Plot the sequences on a number line.

\[
\begin{align*}
3 \frac{1}{2}, &\hspace{0.1cm} 4, &\hspace{0.1cm} 4 \frac{1}{2}, &\hspace{0.1cm} 5, &\hspace{0.1cm} 5 \frac{1}{2}, &\hspace{0.1cm} 6 \\
\frac{13}{4}, &\hspace{0.1cm} \frac{15}{4}, &\hspace{0.1cm} \frac{17}{4}, &\hspace{0.1cm} \frac{19}{4}, &\hspace{0.1cm} \frac{21}{4}, &\hspace{0.1cm} \frac{23}{4} \\
\frac{5}{8}, &\hspace{0.1cm} \frac{5}{8}, &\hspace{0.1cm} \frac{5}{8}, &\hspace{0.1cm} \frac{5}{8}, &\hspace{0.1cm} \frac{3}{8}, &\hspace{0.1cm} \frac{3}{8} \\
\frac{3}{8}, &\hspace{0.1cm} \frac{3}{8}, &\hspace{0.1cm} \frac{3}{8}, &\hspace{0.1cm} \frac{3}{8}, &\hspace{0.1cm} \frac{4}{8}, &\hspace{0.1cm} \frac{4}{8} \\
\end{align*}
\]

Various answers available.

How many ways can you show a difference of one quarter on the number line?

Which sequence is the odd one out? Explain why.

Can you think of a reason why each of the sequences could be the odd one out?

Children may choose different sequences for different reasons. First sequence: the only one containing 6 or it is the only one containing whole numbers. Second sequence: only one using improper fractions. Third sequence: the only one going backwards. Fourth sequence: only one not counting in halves.
Block 3 – Fractions

Theme 2 – Improper fractions and mixed numbers
Improper to Mixed Numbers

Notes and Guidance

Children convert improper fractions to mixed numbers for the first time. An improper fraction is a fraction where the numerator is greater than the denominator. A mixed number is a number consisting of an integer and a proper fraction.

It is important for children to see this process represented visually to allow them to make the connections between the concept and what happens in the abstract.

Mathematical Talk

How many parts are there in a whole?

What do you notice happens to the mixed number when the denominator increases and the numerator remains the same?

What happens when the numerator is a multiple of the denominator?

Varied Fluency

Whitney converts the improper fraction $\frac{14}{5}$ into a mixed number using cubes.
She groups the cubes into 5s, then has 4 left over.

$\frac{5}{5}$ is the same as $\frac{10}{5}$ is the same as

$\frac{14}{5}$ as a mixed number is

Use Whitney’s method to convert $\frac{11}{3}$, $\frac{11}{4}$, $\frac{11}{5}$ and $\frac{11}{6}$

Tommy converts the improper fraction $\frac{27}{8}$ into a mixed number using bar models.

$\frac{27}{8}$

Use Tommy’s method to convert $\frac{25}{8}$, $\frac{27}{6}$, $\frac{18}{7}$ and $\frac{32}{4}$
Amir says, \[ \frac{28}{3} \text{ is less than } \frac{37}{5} \text{ because } 28 \text{ is less than } 37 \]

Do you agree? Explain why.

Possible answer

I disagree because \(\frac{28}{3}\) is equal to \(9 \frac{1}{3}\) and \(\frac{37}{5}\) is equal to \(7 \frac{2}{5}\)

\[ \frac{37}{5} < \frac{28}{3} \]

Spot the mistake

- \(\frac{27}{5} = 5 \frac{2}{5}\)
- \(\frac{27}{3} = 8\)
- \(\frac{27}{4} = 5 \frac{7}{4}\)
- \(\frac{27}{10} = 20 \frac{7}{10}\)

What mistakes have been made?

Can you find the correct answers?

Correct answers

- \(5 \frac{2}{5}\) (incorrect number of fifths)
- \(9\) (incorrect whole)
- \(6 \frac{3}{4}\) (still have an improper fraction)
- \(2 \frac{7}{10}\) (incorrect number of wholes)
Mixed Numbers to Improper

Notes and Guidance

Children now convert from mixed numbers to improper fractions using concrete and pictorial methods to understand the abstract method.

Ensure children always write their working alongside the concrete and pictorial representations so they can see the clear links to the abstract.

Mathematical Talk

How many quarters/halves/eighths/fifths are there in a whole?

How does multiplication support us in converting from mixed numbers to improper fractions?

Can you explain the steps in converting an improper fraction to a mixed number? Use the vocabulary: numerator, denominator, multiply, add

How could we use the previous bar model to help?

Varied Fluency

Whitney converts $3\frac{2}{5}$ into an improper fraction using cubes.

$\frac{2}{5}$ 1 whole is equal to $\square$ fifths.

$\frac{2}{5}$ 3 wholes are equal to $\square$ fifths.

$\square$ fifths + two fifths = $\square$ fifths

Use Whitney's method to convert $2\frac{2}{3}$, $2\frac{2}{4}$, $2\frac{2}{5}$ and $2\frac{2}{6}$

Jack uses bar models to convert a mixed number into an improper fraction.

$\frac{3}{5}$ wholes + $\square$ fifths

$\square$ fifths + $\square$ fifths = $\square$ fifths

Use Jack's method to convert $2\frac{1}{6}$, $4\frac{1}{6}$, $4\frac{1}{3}$, and $8\frac{2}{3}$
Mixed Numbers to Improper

Reasoning and Problem Solving

Three children have incorrectly converted $3 \frac{2}{5}$ into an improper fraction.

Annie has multiplied the numerator and denominator by 3.

Mo has multiplied the correctly but then forgotten to add on the extra 2 parts.

Dexter has just placed 3 in front of the numerator.

What mistake has each child made?

Fill in the missing numbers.

How many different possibilities can you find for each equation?

$2 \frac{?}{8} = \frac{?}{8}$

$2 \frac{?}{5} = \frac{?}{5}$

Compare the number of possibilities you found.

Teacher notes: Encourage children to make generalisations that the number of solutions is one less than the denominator.
Block 3 – Fractions

Theme 3 – Counting in fractions
Number Sequences

Notes and Guidance

Children count up and down in a given fraction. They continue to use visual representations to help them explore number sequences.

Children also find missing fractions in a sequence and determine whether the sequence is increasing or decreasing and by how much.

Mathematical Talk

What are the intervals between the fractions?

Are the fractions increasing or decreasing?

How much are they increasing or decreasing by?

Can you convert the mixed numbers to improper fractions?

Does this make it easier to continue the sequence?

Varied Fluency

Use the counting stick to count up and down in these fractions.

- Start at 0 and count up in steps of $\frac{1}{4}$
- Start at 4 and count down in steps of $\frac{1}{3}$
- Start at 1 and count up in steps of $\frac{2}{3}$

Complete the missing values on the number line.

Complete the sequences.

\[
\frac{3}{4}, \frac{1}{4}, \frac{3}{4}, \frac{2}{4} \quad \frac{3\frac{1}{3}}{3}, \frac{2\frac{2}{3}}{3}
\]

\[
\frac{5\frac{1}{2}}{5}, \frac{7}{10}, \frac{9}{10} \quad \frac{3}{5}, \frac{3}{5}, \frac{3}{5}
\]
Number Sequences

Reasoning and Problem Solving

Three children are counting in quarters.

Whitney

They are all correct, they are all counting in quarter.
Teddy has simplified all answers and Eva has converted improper fractions to mixed numbers.

Teddy

Who is counting correctly? Explain your reasons.

Eva

Play the fraction game for four players.
Place the four fraction cards on the floor.
Each player stands in front of a fraction.
We are going to count up in tenths starting at 0
When you say a fraction, place your foot on your fraction.

Children can make four tenths by stepping on one tenth and three tenths at the same time.
With one foot, they can count up to 11 tenths or one and one tenth.
With two feet they can count up to 22 tenths.

How can we make 4 tenths?
What is the highest fraction we can count to?
How about if we used two feet?
Compare & Order (Less than 1)

Notes and Guidance

Children build on their equivalent fraction knowledge to compare and order fractions less than 1 where the denominators are multiples of the same number.

Children compare the fractions by finding a common denominator or a common numerator. They use bar models to support their understanding.

Mathematical Talk

How does a bar model help us to visualise the fractions? Should both of our bars be the same size? Why? What does this show us?

If the numerators are the same, how can we compare our fractions?

If the denominators are the same, how can we compare our fractions?

Do we always have to find a common denominator? Can we find a common numerator?

Varied Fluency

Use bar models to compare $\frac{5}{8}$ and $\frac{3}{4}$.

Use this method to help you compare: $\frac{5}{6}$ and $\frac{2}{3}$, $\frac{2}{3}$ and $\frac{5}{9}$, $\frac{7}{16}$ and $\frac{3}{8}$.

Use common numerators to help you compare $\frac{2}{5}$ and $\frac{2}{3}$.

Use this method to help you compare: $\frac{6}{7}$ and $\frac{6}{8}$, $\frac{4}{9}$ and $\frac{4}{5}$, $\frac{4}{11}$ and $\frac{2}{5}$.

Order the fractions from greatest to smallest: $\frac{3}{7}$, $\frac{3}{5}$ and $\frac{3}{8}$, $\frac{2}{3}$, $\frac{5}{11}$, and $\frac{7}{12}$, $\frac{6}{11'}$ and $\frac{2}{3}$. 
Ron makes $\frac{3}{4}$ and $\frac{3}{8}$ out of cubes.

He thinks that $\frac{3}{8}$ is equal to $\frac{3}{4}$

Do you agree? Explain your answer.

Possible answer: I disagree with Ron because the two wholes are not equal. He could have compared using numerators or converted $\frac{3}{4}$ to $\frac{6}{8}$. If he does this he will see that $\frac{3}{4}$ is greater. Children may use bar models or cubes to show this.

Always, sometimes, never?

If one denominator is a multiple of the other you can simplify the fraction with the larger denominator to make the denominators the same.

Example:

Could $\frac{7}{4}$ and $\frac{7}{12}$ be simplified to $\frac{7}{4}$ and $\frac{7}{4}$?

Prove it.

Sometimes

It does not work for some fractions e.g. $\frac{8}{15}$ and $\frac{3}{5}$

But does work for others e.g. $\frac{1}{4}$ and $\frac{9}{12}$
**Compare & Order (More than 1)**

**Notes and Guidance**

Children use their knowledge of ordering fractions less than 1 to help them compare and order fractions greater than 1.

They use their knowledge of common denominators to help them.

Children will compare both improper fractions and mixed numbers during this step.

**Mathematical Talk**

How can we represent the fractions?

How does the bar help us see which fraction is the greatest?

Can we use our knowledge of multiples to help us?

Can you predict which fractions will be greatest? Explain how you know.

Is it more efficient to compare using numerators or denominators?

**Varied Fluency**

- **Use bar models to compare** \( \frac{7}{6} \) **and** \( \frac{5}{3} \)

- **Use this method to help you compare:**
  - \( \frac{5}{2} \) and \( \frac{9}{4} \)
  - \( \frac{11}{6} \) and \( \frac{5}{3} \)
  - \( \frac{9}{4} \) and \( \frac{17}{8} \)

- **Use a bar model to compare** \( 1 \frac{2}{3} \) **and** \( 1 \frac{5}{6} \)

- **Use this method to help you compare:**
  - \( 1 \frac{3}{4} \) and \( 1 \frac{3}{8} \)
  - \( 1 \frac{5}{8} \) and \( 1 \frac{1}{2} \)
  - \( 2 \frac{3}{7} \) and \( 2 \frac{9}{14} \)

- **Order the fractions from greatest to smallest using common denominators:**
  - \( \frac{8}{5} \), \( \frac{11}{10} \), and \( \frac{17}{20} \)
  - \( \frac{?}{20} \), \( \frac{?}{20} \), and \( \frac{?}{20} \)
  - \( 1 \frac{2}{3}, 1 \frac{7}{24} \), and \( \frac{11}{12} \)
Eva and Alex each have two identical pizzas.

Eva says,

I have cut each pizza into 6 equal pieces and eaten 8

Who ate the most pizza?

Use a drawing to support your answer.

Alex ate the most pizza because $\frac{15}{9}$ is greater than $\frac{8}{6}$

Alex says,

I have cut each pizza into 9 equal pieces and eaten 15

Dora looks at the fractions $1\frac{7}{12}$ and $1\frac{3}{4}$

She says,

$1\frac{7}{12}$ is greater than $1\frac{3}{4}$ because the numerator is larger

Do you agree?

Explain why using a model.

Possible answer: I do not agree because $1\frac{3}{4}$ is equivalent to $1\frac{9}{12}$ and this is greater than $1\frac{7}{12}$
Compare & Order (Denominator)

Notes and Guidance

Children use their knowledge of equivalent fractions to compare fractions where the denominators are not multiples of the same number. They find the lowest common multiple of the denominators in order to find equivalent fractions with the same denominators. Children then compare the numerators to find the larger or smaller fraction. Encourage children to also use their number sense to visualise the size of the fractions before converting.

Mathematical Talk

When I know the lowest common multiple, how do I know what to multiply the numerator and denominator by to find the correct equivalent fraction? How is comparing mixed numbers different to comparing proper fractions? Do I need to compare the whole numbers? Why? If the whole numbers are the same, what do I do? Can you plot the fractions on a number line to estimate which is the smallest? Which fractions are larger/smaller than a half? How does this help me order the fractions?

Varied Fluency

- Use the bar models to compare $\frac{3}{4}$ and $\frac{2}{3}$.
  - ___ is greater than ___
  - ___ is less than ___

- Dora is comparing $\frac{5}{6}$ and $\frac{3}{4}$ by finding the lowest common multiple of the denominators.
  - Multiples of 6: 6, 12, 18, 24
  - Multiples of 4: 4, 8, 12, 16
  - 12 is the LCM of 4 and 6
  - \[
  \frac{5}{6} = \frac{10}{12}, \quad \frac{3}{4} = \frac{9}{12}
  \]
  - \[
  \frac{10}{12} > \frac{9}{12}
  \]

- Use Dora’s method to compare the fractions.
  - $\frac{4}{5}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{3}{4}$, $\frac{7}{10}$, $\frac{2}{5}$, $\frac{3}{8}$

- Order the fractions in descending order.
  - $\frac{3}{8}$, $\frac{11}{20}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{10}$
  - Which fraction is the greatest?
  - Which fraction is the smallest?
Compare & Order (Denominator)

Reasoning and Problem Solving

Use the digit cards to complete the statements.

5 6 3 4

\[
\frac{5}{4} > \frac{3}{6}
\]

\[
\frac{3}{4} < \frac{6}{5} \text{ or } \frac{5}{4} < \frac{6}{3}
\]

Find three examples of ways you could complete the statement.

[Blank] [Blank] [Blank] [6]

\[
\frac{4}{6} < \frac{6}{4}
\]

Is Teddy correct? Explain why.

Teddy is comparing \(\frac{3}{8}\) and \(\frac{5}{12}\)

To find the lowest common multiple, I will multiply 8 and 12 together.

\[8 \times 12 = 96\]

I will use a common denominator of 96.

Teddy is incorrect because the LCM of 8 and 12 is 24. 96 is a common multiple so he would still compare the fractions correctly but it is not the most efficient method.

More answers available.
Compare & Order (Numerator)

Notes and Guidance

Building on their prior knowledge of comparing unit fractions, children look at comparing fractions by finding a common numerator. They focus on the idea that when the numerators are the same, the larger the denominator, the smaller the fraction.

Children consider the most efficient method when comparing fractions and decide whether to find common numerators or common denominators.

Mathematical Talk

What’s the same and what’s different about the fractions on the bar models? How can we compare them? Can you use the words greatest and smallest to complete the sentences?

Do you need to change one or both numerators? Why?

How can you decide whether to find a common numerator or denominator?

Varied Fluency

Compare the fractions.

When the denominators are the same, the _____ the numerator, the _____ the fraction.

When the numerators are the same, the ______ the denominator, the ______ the fraction.

Jack is comparing $\frac{2}{5}$ and $\frac{4}{7}$ by finding the LCM of the numerators.

The LCM of 2 and 4 is 4.

Use Jack's method to compare the fractions.
Mo is comparing the fractions $\frac{3}{7}$ and $\frac{6}{11}$.

He wants to find a common denominator.

Explain whether you think this is the most effective strategy.

This is not the most effective strategy because both denominators are prime. He could find a common numerator by changing $\frac{3}{7}$ into $\frac{6}{14}$ and comparing them by using the rule ‘when the numerator is the same, the smaller the denominator, the bigger the fraction’ $\frac{6}{11}$ is bigger.

Two different pieces of wood have had a fraction chopped off.

Here are the pieces now, with the fraction that is left.

Which piece of wood was the longest to begin with?

Explain your answer.

Can you explain your method?

The second piece was longer because $\frac{1}{4}$ is greater than $\frac{1}{6}$. Children can explain their methods and how they compared one quarter and one sixth.
Add & Subtract Fractions

Notes and Guidance

Children recap their Year 4 understanding of adding and subtracting fractions with the same denominator.

They use bar models to support understanding of adding and subtracting fractions.

Mathematical Talk

How many equal parts do I need to split my bar into?

Can you convert the improper fraction into a mixed number?

How can a bar model help you balance both sides of the equals sign?

Varied Fluency

Here is a bar model to calculate $\frac{3}{5} + \frac{4}{5}$

$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

Use a bar model to solve the calculations:

$$\frac{3}{8} + \frac{3}{8} \quad \frac{5}{6} + \frac{1}{6} \quad \frac{5}{3} + \frac{5}{3}$$

Here are two bar models to calculate $\frac{7}{8} - \frac{3}{8}$

What is the difference between the two methods?

Use your preferred method to calculate:

$$\frac{5}{8} - \frac{1}{8} \quad \frac{9}{7} - \frac{4}{7} \quad \frac{5}{3} - \frac{5}{3} \quad 1 - \frac{2}{5}$$

Calculate:

$$\frac{3}{7} + \frac{5}{7} = \frac{4}{7} \quad \frac{9}{5} - \frac{5}{5} = \frac{6}{5} \quad \frac{2}{3} + \frac{4}{3} = \frac{11}{3} - \frac{4}{3}$$
Add & Subtract Fractions

Reasoning and Problem Solving

How many different ways can you balance the equation?

\[
\frac{5}{9} + \frac{\square}{9} = \frac{8}{9} + \frac{\square}{9}
\]

Possible answers:

\[
\frac{5}{9} + \frac{3}{9} = \frac{8}{9} + \frac{0}{9}
\]

\[
\frac{5}{9} + \frac{4}{9} = \frac{8}{9} + \frac{1}{9}
\]

\[
\frac{5}{9} + \frac{5}{9} = \frac{8}{9} + \frac{2}{9}
\]

Any combination of fractions where the numerators add up to the same total on each side of the equals sign.

A chocolate bar has 12 equal pieces.

Amir eats \(\frac{5}{12}\) more of the bar than Whitney.

There is one twelfth of the bar remaining.

What fraction of the bar does Amir eat?

What fraction of the bar does Whitney eat?

Amir eats \(\frac{8}{12}\) of the chocolate bar and Whitney eats \(\frac{3}{12}\) of the chocolate bar.
Add Fractions within 1

Notes and Guidance

Children add fractions with different denominators for the first time where one denominator is a multiple of the other.

They use pictorial representations to convert the fractions so they have the same denominator.

Ensure children always write their working alongside the pictorial representations so they see the clear links.

Mathematical Talk

Can you find a common denominator? Do you need to convert both fractions or just one?

Can you explain Mo and Rosie's methods to a partner? Which method do you prefer?

How do Mo and Rosie's methods support finding a common denominator?

Varied Fluency

Mo is calculating $\frac{1}{2} + \frac{1}{8}$

He uses a diagram to represent the sum.

\[
\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}
\]

Use Mo's method to solve:

\[
\begin{align*}
\frac{1}{2} + \frac{3}{8} & \quad \frac{1}{4} + \frac{3}{8} & \quad \frac{7}{10} + \frac{1}{5}
\end{align*}
\]

Rosie is using a bar model to solve $\frac{1}{4} + \frac{3}{8}$

\[
\begin{align*}
\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}
\end{align*}
\]

Use a bar model to solve:

\[
\begin{align*}
\frac{1}{6} + \frac{5}{12} & \quad \frac{2}{9} + \frac{1}{3} & \quad \frac{1}{3} + \frac{4}{15}
\end{align*}
\]
Add Fractions within 1

Reasoning and Problem Solving

\[ \frac{5}{16} + \square = \frac{15}{16} \]

5

\[ \square + \frac{7}{10} = \frac{17}{20} \]

3

Two children are solving \( \frac{1}{3} + \frac{4}{15} \)

Eva starts by drawing this model:

 Possible answer: Each child may have started with a different fraction in the calculation.
E.g. Eva has started by shading a third. She now needs to divide each third into five equal parts so there are fifteen equal parts altogether. Eva will then shade \( \frac{4}{15} \) and will have \( \frac{9}{15} \) altogether.

Annie solved this calculation.

\[ \frac{3}{4} + \frac{3}{16} = \frac{3 + 3}{4 + 16} = \frac{6}{20} = \frac{3}{10} \]

Annie is wrong because she has just added the numerators and the denominators. When adding fractions with different denominators you need to find a common denominator.

Can you spot and explain her mistake?

Can you explain each person’s method and how they would complete the question?
Which method do you prefer and why?
Year 5 | Spring Term | Week 4 to 9 – Number: Fractions

Add 3 or More Fractions

Notes and Guidance

Children add more than 2 fractions where two denominators are a multiple of the other.

They use a bar model to continue exploring this.

Ensure children always write their working alongside the pictorial representations so they see the clear links.

Mathematical Talk

Can you find a common denominator? Do you need to convert both fractions or just one?

Can you explain Ron's method to a partner? How does Ron's method support finding a common denominator?

Can you draw what Farmer Stanef's field could look like? What fractions could you divide your field into?

Why would a bar model not be efficient for this question?

Varied Fluency

Ron uses a bar model to calculate \( \frac{2}{5} + \frac{1}{10} + \frac{3}{20} \)

Use a bar model to solve:

\( \frac{1}{4} + \frac{3}{8} + \frac{5}{16} \)  \( \frac{1}{2} + \frac{1}{6} + \frac{1}{12} \)

Farmer Stanef owns a field.  
He plants carrots on \( \frac{1}{3} \) of the field.  
He plants potatoes on \( \frac{2}{9} \) of the field.  
He plants onions on \( \frac{5}{18} \) of the field.  
What fraction of the field is covered altogether?

Complete the fractions.

\( \frac{1}{5} + \boxed{\ } + \frac{8}{20} = 1 \)  \( \frac{1}{5} + \boxed{\ } + \frac{1}{30} = 1 \)
Add 3 or More Fractions

Reasoning and Problem Solving

Eva is attempting to answer:

\[
\frac{3}{5} + \frac{1}{10} + \frac{3}{20}
\]

Eva is wrong because she has added the numerators and denominators together and hasn’t found a common denominator. The correct answer is

\[
\frac{7}{35}
\]

Do you agree with Eva? Explain why.

Jack has added 3 fractions together to get an answer of \(\frac{17}{18}\)

What 3 fractions could he have added?

Can you find more than one answer?

Possible answers:

\[
\frac{1}{18} + \frac{4}{18} + \frac{13}{18}
\]

\[
\frac{1}{9} + \frac{5}{9} + \frac{5}{18}
\]

\[
\frac{1}{6} + \frac{5}{9} + \frac{2}{9}
\]

\[
\frac{1}{18} + \frac{1}{6} + \frac{13}{18}
\]

\[
\frac{1}{3} + \frac{1}{6} + \frac{4}{9}
\]
Add Fractions

Notes and Guidance

Children continue to represent adding fractions using pictorial methods to explore adding two or more proper fractions where the total is greater than 1.

Children can record their totals as an improper fraction but will then convert this to a mixed number using their prior knowledge.

Mathematical Talk

How does the pictorial method support me to add the fractions?

Which common denominator will we use?

How do my times-tables support me to add fractions?

Which representation do you prefer? Why?

Varied Fluency

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Fraction Model 1]</td>
<td>![Fraction Model 2]</td>
<td>![Fraction Model 3]</td>
</tr>
</tbody>
</table>

\[
\frac{1}{3} + \frac{5}{6} + \frac{5}{12} = 1 \frac{7}{12}
\]

Explain each step of the calculation.

Use this method to help you add the fractions.

Give your answer as a mixed number.

\[
\frac{2}{3} + \frac{1}{6} + \frac{7}{12} = \frac{1}{4} + \frac{7}{8} + \frac{3}{16}
\]

\[
\frac{1}{2} + \frac{5}{6} + \frac{5}{12}
\]

Use the bar model to add the fractions. Record your answer as a mixed number.

\[
\frac{3}{4} + \frac{3}{8} + \frac{1}{2} =
\]

Draw your own models to solve:

\[
\frac{5}{12} + \frac{1}{6} + \frac{1}{2} = \frac{11}{20} + \frac{3}{5} + \frac{1}{10} = \frac{3}{4} + \frac{5}{12} + \frac{1}{2}
\]
Add Fractions

Reasoning and Problem Solving

Annie is adding three fractions. She uses the model to help her.

What could her three fractions be?

How many different combinations can you find?

Can you write a number story to represent your calculation?

Possible answer:
\[
\frac{2}{3} + \frac{4}{12} + \frac{1}{2} = 1\frac{1}{2}
\]

Other equivalent fractions may be used.

Example story:
Some children are eating pizzas. Jack eats two thirds, Amir eats four twelfths and Dexter eats half a pizza. How much pizza did they eat altogether?

The sum of three fractions is \(2\frac{1}{8}\)

The fractions have different denominators.

All of the fractions are greater than or equal to a half.

None of the fractions are improper fractions.

All of the denominators are factors of 8

What could the fractions be?

\[
\frac{1}{2} + \frac{3}{4} + \frac{7}{8}
\]

Children could be given less clues and explore other possible solutions.
Add Mixed Numbers

Notes and Guidance

Children move on to adding two fractions where one or both are mixed numbers or improper fractions.

They will use a method of adding the wholes and then adding the parts. Children will record their answer in its simplest form.

Children can still draw models to represent adding fractions.

Mathematical Talk

How can we partition these mixed numbers into whole numbers and fractions?

What will the wholes total? Can I add the fractions straight away?

What will these mixed numbers be as improper fractions?

If I have an improper fraction in the question, should I change it to a mixed number first? Why?

Varied Fluency

Add the fractions by adding the whole first and then the fractions. Give your answer in its simplest form.

\[ 1 \frac{1}{3} + 2 \frac{1}{6} = 3 + \frac{3}{6} = 3 \frac{3}{6} \text{ or } 3 \frac{1}{2} \]

\[ \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} \]

Add the fractions by converting them to improper fractions.

\[ 1 \frac{3}{4} + 2 \frac{3}{8} = \frac{7}{4} + \frac{17}{8} = \frac{14}{8} + \frac{17}{8} = \frac{31}{8} = 3 \frac{7}{8} \]

Add these fractions.

\[ 1 \frac{1}{4} + 2 \frac{5}{12} \quad 2 \frac{1}{9} + 1 \frac{1}{3} \quad 2 \frac{1}{6} + 2 \frac{2}{3} \]

\[ 4 \frac{7}{9} + 2 \frac{1}{3} \quad \frac{17}{6} + 1 \frac{1}{3} \quad \frac{15}{8} + 2 \frac{1}{4} \]

How do they differ from previous examples?
Add Mixed Numbers

Reasoning and Problem Solving

Jack and Whitney have some juice.

Jack drinks $2\frac{1}{4}$ litres and Whitney drinks $2\frac{5}{12}$ litres.

How much do they drink altogether?

Complete this using two different methods.

Which method do you think is more efficient? Why?

They drink $4\frac{2}{3}$ litres altogether.

Encourage children to justify which method they prefer and why. Ensure children discuss which method is more or less efficient.

Fill in the missing numbers.

$4 \quad \frac{5}{6} \quad + \quad \square \quad \square \quad = \quad 10 \quad \frac{1}{3}$

$5\frac{3}{6}$ or $5\frac{1}{2}$
Subtract Fractions

Notes and Guidance

Children subtract fractions with different denominators for the first time, where one denominator is a multiple of the other.

It is important that subtraction is explored as both take away and finding the difference.

Mathematical Talk

What could the common denominator be?

Can you draw a model to help you solve the problem?

Is it easier to use a take away bar model (single bar model) or a bar model to find the difference (comparison model)?

Varied Fluency

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{3})</td>
<td>(\frac{4}{12})</td>
<td>(\frac{1}{3} - \frac{1}{12} = \frac{3}{12})</td>
</tr>
</tbody>
</table>

Explain each step of the calculation.

Use this method to help you solve \(\frac{5}{6} - \frac{1}{3}\) and \(\frac{7}{8} - \frac{5}{16}\).

Tommy and Teddy both have the same sized chocolate bar. Tommy has \(\frac{3}{4}\) left, Teddy has \(\frac{5}{12}\) left. How much more does Tommy have?

Amir uses a number line to find the difference between \(\frac{5}{9}\) and \(\frac{4}{3}\).

Use this method to find the difference between:

\(\frac{3}{4}\) and \(\frac{5}{12}\)

\(\frac{19}{15}\) and \(\frac{3}{5}\)

\(\frac{20}{9}\) and \(\frac{4}{3}\)
Subtract Fractions

Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Which subtraction is the odd one out?</th>
<th>Possible answers:</th>
</tr>
</thead>
</table>
| A  \[
\frac{13}{4} - \frac{3}{8}
\]  |
| C  \[
\frac{23}{7} - \frac{1}{3}
\]  |
| B  \[
\frac{10}{9} - \frac{2}{9}
\]  |

C is the odd one out because the denominators aren’t multiples of each other.

A is the odd one out because the denominators are even.

B is the odd one out because it is the only answer above 3

The perimeter of the rectangle is \[\frac{16}{9}\]

Work out the missing length.

The missing length is \[\frac{2}{3}\]
Subtract Mixed Numbers (1)

Notes and Guidance

Children apply their understanding of subtracting fractions where one denominator is a multiple of the other to subtract proper fractions from mixed numbers.

They continue to use models and number lines to support their understanding.

Mathematical Talk

Which fraction is the greatest? How do you know?

If the denominators are different, what can we do?

Can you simplify your answer?

Which method do you prefer when subtracting fractions: taking away or finding the difference?

Varied Fluency

Use this method to help you solve:

\[ 2 \frac{3}{5} - \frac{3}{10} \quad 1 \frac{2}{3} - \frac{1}{6} \quad 1 \frac{5}{6} - \frac{7}{12} \]

Use a number line to find the difference between \(1 \frac{2}{5}\) and \(\frac{3}{10}\):

\[ 1 \frac{2}{5} = 1 \frac{4}{10} \]

Use a number line to find the difference between:

\[ 3 \frac{5}{6} \text{ and } \frac{1}{12} \quad 5 \frac{5}{7} \text{ and } \frac{3}{14} \quad 2 \frac{7}{9} \text{ and } \frac{11}{18} \]

Solve:

\[ 1 \frac{2}{3} - \frac{5}{6} \quad 1 \frac{3}{4} - \frac{7}{8} \quad 2 \frac{3}{8} - \frac{11}{16} \]
Subtract Mixed Numbers (1)

Reasoning and Problem Solving

Amir is attempting to solve \(2 \frac{5}{14} - \frac{2}{7}\)

Here is his working out:

\[
2 \frac{5}{14} - \frac{2}{7} = 2 \frac{3}{7}
\]

Do you agree with Amir? Explain your answer.

Possible answer:
Amir is wrong because he hasn’t found a common denominator when subtracting the fractions he has just subtracted the numerators and the denominators. The correct answer is \(2 \frac{1}{14}\).

Here is Rosie’s method. What is the calculation?

Can you find more than one answer? Why is there more than one answer?

The calculation could be \(1 \frac{5}{6} - \frac{7}{12}\) or \(1 \frac{10}{12} - \frac{7}{12}\).

There is more than one answer because five sixths and ten twelfths are equivalent. Children should be encouraged to write the question as \(1 \frac{5}{6} - \frac{7}{12}\) so that all fractions are in their simplest form.
Subtract Mixed Numbers (2)

Notes and Guidance

Children use prior knowledge of fractions to subtract two fractions where one is a mixed number and you need to break one of the wholes up.

They use the method of flexible partitioning to create a new mixed number so they can complete the calculation.

Mathematical Talk

Is flexible partitioning easier than converting the mixed number to an improper fraction?

Do we always have to partition the mixed number?

When can we subtract a fraction without partitioning the mixed number in a different way?

Varied Fluency

We can work out $2 \frac{3}{4} - \frac{7}{8}$ using this method.

Use this method to calculate:

- $3 \frac{1}{3} - \frac{5}{6}$
- $4 \frac{1}{5} - \frac{7}{10}$
- $5 \frac{2}{3} - \frac{4}{9}$

Use flexible partitioning to solve $7 \frac{1}{3} - \frac{5}{6}$

$7 \frac{1}{3} - \frac{5}{6} = 6 + 1 \frac{1}{3} - \frac{5}{6} = 6 + 1 \frac{2}{6} - \frac{5}{6} = 6 \frac{3}{6} = 6 \frac{1}{2}$

Use this method to calculate:

- $4 \frac{2}{3} - \frac{5}{6}$
- $4 \frac{1}{5} - \frac{7}{15}$
- $5 \frac{1}{4} - \frac{7}{8}$

Mr Brown has $3 \frac{1}{4}$ bags of flour. He uses $\frac{7}{8}$ of a bag. How much flour does he have left?
Subtract Mixed Numbers (2)

Reasoning and Problem Solving

Place 2, 3 and 4 in the boxes to make the calculation correct.

\[
27 \boxed{\frac{1}{3}} - \boxed{\frac{4}{6}} = 26 \boxed{\frac{2}{3}}
\]

3 children are working out \[6 \frac{2}{3} - \frac{5}{6}\]

They partition the mixed number in the following ways to help them.

- **Dora**
  \[
  5 + 1 \frac{2}{3} - \frac{5}{6}
  \]

- **Alex**
  \[
  5 + 1 \frac{4}{6} - \frac{5}{6}
  \]

- **Jack**
  \[
  5 + \frac{10}{6} - \frac{5}{6}
  \]

Are they all correct?
Which method do you prefer?
Explain why.

All three children are correct.

\[1\frac{2}{3}, 1\frac{4}{6} \text{ and } 1\frac{10}{6} \] are all equivalent therefore all three methods will help children to correctly calculate the answer.
Subtract 2 Mixed Numbers

Notes and Guidance

Children use different strategies to subtract two mixed numbers.

Building on learning in previous steps, they look at partitioning the mixed numbers into wholes and parts and build on their understanding of flexible partitioning as well as converting to improper fractions when an exchange is involved.

Mathematical Talk

Why is subtracting the wholes and parts separately easier with some fractions than others?

Can you show the subtraction as a difference on a number line? Bar model? How are these different to taking away?

Does making the whole numbers larger make the subtraction any more difficult? Explain why.

Varied Fluency

Here is a bar model to calculate $3\frac{5}{8} - 2\frac{1}{4}$

$3 - 2 = 1$

Use this method to calculate:

$3\frac{7}{8} - 2\frac{3}{4}$

$5\frac{5}{6} - 2\frac{1}{3}$

$3\frac{8}{9} - 2\frac{5}{27}$

Why does this method not work effectively for $5\frac{1}{6} - 2\frac{1}{3}$?

Here is a method to calculate $5\frac{1}{6} - 2\frac{1}{3}$

Use this method to calculate:

$3\frac{1}{4} - 2\frac{5}{8}$

$5\frac{1}{3} - 2\frac{7}{12}$

$27\frac{1}{3} - 14\frac{7}{15}$
### Subtract 2 Mixed Numbers

#### Reasoning and Problem Solving

| There are three colours of dog biscuits in a bag of dog food: red, brown and orange. | 3 $\frac{3}{4} + 1 \frac{7}{16} = 5 \frac{3}{16}$ |
| The total mass of the dog food is 7 kg. | 7 $- 5 \frac{3}{16} = 1 \frac{13}{16}$ |
| The mass of red biscuits is $3 \frac{3}{4}$ kg and the mass of the brown biscuits is $1 \frac{7}{16}$ kg. | The mass of orange biscuits is $1 \frac{13}{16}$ kg. |
| What is the mass of orange biscuits? | Rosie has 20 $\frac{3}{4}$ cm of ribbon. |
| | Annie has 6 $\frac{7}{8}$ cm less ribbon than Rosie. |
| | How much ribbon does Annie have? |
| | How much ribbon do they have altogether? |
| | Annie has 13 $\frac{7}{8}$ cm of ribbon. |
| | Altogether they have 34 $\frac{5}{8}$ cm of ribbon. |
Add & Subtract Fractions (1)

Notes and Guidance

Children add and subtract fractions within 1 where the denominators are multiples of the same number. Encourage children to find the lowest common multiple in order to find a common denominator. Ensure children are confident with the understanding of adding and subtracting fractions with the same denominator. Bar models can support this, showing children that the denominators stay the same whilst the numerators are added or subtracted.

Mathematical Talk

If the denominators are different, when we are adding or subtracting fractions, what do we need to do? Why?

How does finding the lowest common multiple help to find a common denominator?

Can you use a bar model to represent Eva’s tin of paint? On which day did Eva use the most paint? On which day did Eva use the least paint? How much more paint did Eva use on Friday than Saturday?

Varied Fluency

Whitney is calculating $\frac{5}{8} + \frac{3}{16}$

She finds the lowest common multiple of 8 and 16 to find a common denominator.

LCM of 8 and 16 is 16

$\frac{5}{8} = \frac{10}{16}$

$\frac{10}{16} + \frac{3}{16} = \frac{13}{16}$

Use this method to calculate:

$\frac{1}{3} + \frac{2}{9} = \frac{3}{7} + \frac{7}{21} = \frac{8}{15} + \frac{1}{5} = \frac{3}{16} + \frac{3}{8} + \frac{1}{4} = \frac{13}{16}$

Find a common denominator for each pair of fractions by using the lowest common multiple. Subtract the smaller fraction from the larger fraction in each pair.

$\frac{3}{4} - \frac{5}{8} = \frac{7}{12} - \frac{1}{3} = \frac{11}{16} - \frac{3}{4} = \frac{14}{15} - \frac{2}{5} = \frac{8}{9} - \frac{1}{3}$

Eva has a full tin of paint. She uses $\frac{1}{3}$ of the tin on Friday, $\frac{1}{21}$ on Saturday and $\frac{2}{7}$ on Sunday. How much paint does she have left?
Add & Subtract Fractions (1)

Reasoning and Problem Solving

Use the same digit in both boxes to complete the calculation.
Is there more than one way to do it?

\[
\frac{\Box}{20} + \frac{1}{\Box} = \frac{9}{20}
\]

\[
\frac{4}{20} + \frac{1}{4} = \frac{9}{20}
\]

\[
\frac{5}{20} + \frac{1}{5} = \frac{9}{20}
\]

Alex is adding fractions.

\[
\frac{3}{5} + \frac{1}{15} = \frac{4}{20} = \frac{1}{5}
\]

Do you agree with her?
Explain your answer.

Dexter subtracted \( \frac{3}{5} \) from a fraction and his answer was \( \frac{8}{45} \).
What fraction did he subtract \( \frac{3}{5} \) from?
Give your answer in its simplest form.

\[
\frac{8}{45} + \frac{3}{5} = \frac{8}{45} + \frac{27}{45}
\]

\[
\frac{8}{45} + \frac{27}{45} = \frac{35}{45} = \frac{7}{9}
\]

Dexter subtracted \( \frac{3}{5} \) from \( \frac{7}{9} \).

Alex is wrong because she has added the numerators and the denominators rather than finding a common denominator. It should be

\[
\frac{9}{15} + \frac{1}{15} = \frac{10}{15} = \frac{2}{3}
\]
Add & Subtract Fractions (2)

Notes and Guidance

Children add and subtract fractions where the denominators are not multiples of the same number. They continue to find the lowest common multiple, but now need to find equivalent fractions for both fractions in the calculation to find a common denominator.

When the denominators are not multiples of the same number, support children to notice that we need to multiply the denominators together in order to find the LCM.

Mathematical Talk

What is the same about all the subtractions? \(\frac{3}{4}\)

What do you notice about the LCM of all the denominators?

Which of the subtractions has the biggest difference? Explain how you know. Can you order the differences in ascending order?

How can we find the LCM of three numbers? Do we multiply them together? Is 120 the LCM of 4, 5 and 6?

Varied Fluency

Amir is calculating \(\frac{7}{9} - \frac{1}{2}\)

He finds the lowest common multiple of 9 and 2

LCM of 9 and 2 is 18

\[
\frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18}
\]

Use this method to calculate:

\[
\frac{3}{4} - \frac{1}{3} = \frac{3}{4} - \frac{3}{5} = \frac{3}{4} - \frac{2}{7} = \frac{3}{4} - \frac{7}{11} =
\]

Eva has a bag of carrots weighing \(\frac{3}{4}\) kg and a bag of potatoes weighing \(\frac{2}{5}\) kg. She is calculating how much they weigh altogether.

The LCM of 4 and 5 is 20. I will convert the fractions to twentieths.

\[
\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1 \frac{3}{20} \text{ kg}
\]

Use this method to calculate:

\[
\frac{1}{4} + \frac{2}{5} = \frac{7}{8} + \frac{1}{3} = \frac{5}{6} + \frac{5}{7} = \frac{13}{20} + \frac{2}{3} =
\]

On Friday, Ron walks \(\frac{5}{6}\) km to school, \(\frac{3}{4}\) km to the shops and \(\frac{4}{5}\) km home. How far does he walk altogether?
## Add & Subtract Fractions (2)

### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>A car is travelling from Halifax to Brighton.</th>
<th>The car has travelled $\frac{13}{15}$ of the journey altogether.</th>
<th>Mr and Mrs Rose and knitting scarves. Mr Rose's scarf is $\frac{5}{9}$ m long. Mrs Rose's scarf is $\frac{1}{5}$ m longer than Mr Rose's scarf. How long is Mrs Rose's scarf? How long are both the scarves altogether?</th>
<th>Mrs Rose's scarf is $\frac{34}{45}$ m long. Both scarves together are $1\frac{14}{45}$ m long.</th>
</tr>
</thead>
<tbody>
<tr>
<td>In the morning, it completes $\frac{2}{3}$ of the journey. In the afternoon, it completes $\frac{1}{5}$ of the journey. What fraction of the journey has been travelled altogether? What fraction of the journey is left to travel?</td>
<td>There is $\frac{2}{15}$ of the journey left to travel.</td>
<td>Fill in the boxes to make the calculation correct. Various answers available. E.g. $1\frac{1}{10} = \frac{3}{5} + \frac{5}{10}$.</td>
<td></td>
</tr>
<tr>
<td>If the journey is 270 miles, how far did the car travel in the morning? How far did the car travel in the afternoon? How far does the car have left to travel?</td>
<td>The car travelled 180 miles in the morning. The car travelled 54 miles in the afternoon. The car has 36 miles left to travel.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Add Fractions

Notes and Guidance

Children explore adding mixed numbers. They look at different methods depending on whether the fractions total more than one. They add fractions with any denominators, building on their understanding from the previous steps. Encourage children to draw bar models to support them in considering whether the fractions will cross the whole. They continue to simplify answers and convert between improper fractions and whole numbers when calculating.

Mathematical Talk

How many wholes are there altogether?
Can you find the LCM of the denominators to find a common denominator?
Do you prefer Tommy or Whitney’s method? Why?
Does Tommy’s method work when the fractions add to more than one? How could we adapt his method?
Does Whitney’s method work effectively when there are large whole numbers?

Varied Fluency

Tommy is adding mixed numbers. He adds the wholes and then adds the fractions. Then, Tommy simplifies his answer.

\[
1 \frac{1}{2} + 2 \frac{1}{6} = 1 \frac{3}{6} + 2 \frac{1}{6} = 3 \frac{4}{6} = 3 \frac{2}{3}
\]

Use Tommy’s method to add the fractions.

\[
3 \frac{1}{2} + 2 \frac{3}{8} = \quad 34 \frac{1}{9} + 5 \frac{2}{5} = \quad 12 \frac{5}{12} + 2 \frac{1}{7} =
\]

Whitney is also adding mixed numbers. She converts them to improper fractions, adds them, and then converts them back to a mixed number.

\[
1 \frac{1}{2} + 2 \frac{1}{6} = \frac{3}{2} + \frac{13}{6} = \frac{9}{6} + \frac{13}{6} = \frac{22}{6} = 3 \frac{4}{6} = 3 \frac{2}{3}
\]

Use Whitney’s method to add the fractions.

\[
3 \frac{1}{2} + 2 \frac{3}{8} \quad 2 \frac{1}{9} + 2 \frac{2}{5} \quad 2 \frac{7}{9} + 2 \frac{2}{5} \quad 4 \frac{3}{4} + 3 \frac{11}{15}
\]

Jug A has 2 \(\frac{3}{4}\) litres of juice in it. Jug B has 3 \(\frac{4}{5}\) litres of juice in it. How much juice is there in Jug A and Jug B altogether?
Add Fractions

Reasoning and Problem Solving

Each row and column adds up to make the total at the end. Use this information to complete the diagram.

Dora is baking muffins. She uses \(2 \frac{1}{4}\) kg of flour, \(1 \frac{3}{5}\) kg of sugar and \(1 \frac{1}{4}\) kg of butter.

How much flour, sugar and butter does she use altogether?

How much more flour does she use than butter?

How much less butter does she use than sugar?

Dora uses \(5 \frac{7}{20}\) kg of flour, sugar and butter altogether.

Dora uses \(1 \frac{1}{4}\) kg more flour than butter.

Dora uses \(\frac{7}{20}\) kg less butter than sugar.
Subtract Fractions

Notes and Guidance

Children subtract mixed numbers. They explore different methods including exchanging wholes for fractions and subtracting the wholes and fractions separately and converting the mixed number to an improper fraction.

Encourage children to consider which method is the most efficient depending on the fractions they are subtracting. Bar models can support to help children to visualise the subtraction and understand the procedure.

Mathematical Talk

How many eighths can we exchange for one whole?

What is the same about the first set of subtractions?

What is different about the subtractions? (How does this affect the subtraction?)

Do you prefer Annie’s or Amir’s method? Why?

Look at Amir’s calculation, what do you notice about the relationship between \( \frac{2}{5} \) and \( \frac{7}{10} \)? (\( \frac{2}{5} \) is double \( \frac{7}{10} \))

Varied Fluency

Annie is calculating \( 3\frac{1}{4} - 1\frac{3}{4} \)

I can’t subtract the wholes and fractions separately because \( \frac{1}{4} \) is less than \( \frac{3}{4} \). I will exchange 1 whole for 4 quarters. \( 3\frac{1}{4} = 2\frac{5}{4} \)

\[
\begin{align*}
3\frac{1}{4} - 1\frac{3}{4} &= 2\frac{5}{4} - 1\frac{3}{4} = 1\frac{2}{4} = 1\frac{1}{2} \\
3\frac{1}{8} - 1\frac{3}{8} &= 3\frac{1}{8} - 1\frac{1}{2} = 3\frac{1}{8} - 1\frac{1}{5} = 3\frac{1}{8} - 1\frac{3}{5} =
\end{align*}
\]

Use Annie’s method to calculate:

Amir is calculating \( 3\frac{2}{5} - 1\frac{7}{10} \)

He converts the mixed numbers to improper fractions to subtract them.

\[
\begin{align*}
3\frac{2}{5} - 1\frac{7}{10} &= \frac{17}{5} - \frac{17}{10} = \frac{34}{10} - \frac{17}{10} = \frac{17}{10} = 1\frac{7}{10} \\
4\frac{4}{5} - 1\frac{9}{10} &= 2\frac{1}{7} - 1\frac{1}{3} = 3\frac{5}{12} - 1\frac{7}{9} = 3\frac{5}{11} - 1\frac{4}{5} =
\end{align*}
\]
Subtract Fractions

Reasoning and Problem Solving

A blue, orange and green box are on a number line.

The number in the green box is \( \frac{2}{3} \) more than the orange box.

The number in the orange box is: \( \frac{3}{4} \)

The number in the orange box is \( \frac{11}{16} \) greater than the number in the blue box.

\[
5 \frac{5}{12} - 3 \frac{2}{3} = 1 \frac{9}{12}
\]

The orange box is \( \frac{3}{4} \)

\[
1 \frac{3}{4} - 1 \frac{1}{16} = \frac{11}{16}
\]

The orange box is \( \frac{11}{16} \) greater than the blue box.

Complete the part-whole model.

Jack is calculating \( 4 \frac{2}{7} - 2 \frac{6}{7} \)

He adds \( \frac{1}{7} \) to both numbers.

\[
4 \frac{2}{7} - 2 \frac{6}{7} = 4 \frac{3}{7} - 3
\]

so the answer is \( 1 \frac{3}{7} \)

Jack has increased both mixed numbers by \( \frac{1}{7} \) so the difference has remained constant.

Explain why Jack is correct.
Mixed Addition & Subtraction

Notes and Guidance

Children solve problems that involve adding and subtracting fractions and mixed numbers. Encourage children to consider the most efficient method of adding and subtracting fractions and to simplify their answers when possible.

Children can use bar models to represent the problems and support them in deciding whether they need to add or subtract. They can share their different methods to gain a flexible approach to calculating with fractions.

Mathematical Talk

Can you draw a bar model to represent the problem? Do we need to add or subtract the fractions?

How do I know if my answer is simplified fully?

What is the lowest common multiple of the denominators?

How can I calculate the area covered by each vegetable? If you know the area for carrots and cabbages, how can you work out the area for potatoes? Can you think of 2 different ways?

Varied Fluency

Alex has 5 bags of sweets.

On Monday she eats \( \frac{2}{3} \) of a bag and gives \( \frac{4}{5} \) of a bag to her friend.

On Tuesday she eats \( 1 \frac{1}{3} \) bags and gives \( \frac{2}{5} \) of a bag to her friend.

What fraction of her sweets does Alex have left?

Give your answer in its simplest form.

Here is a vegetable patch. \( \frac{1}{5} \) of the patch is for carrots. \( \frac{3}{8} \) of the patch is for cabbages.

What fraction of the patch is for carrots and cabbages altogether?

What fraction of the patch is for potatoes?

What fraction more of the patch is for potatoes than cabbages?

Give your answers in their simplest form.

The vegetable patch has an area of 80 m\(^2\).

What is the area covered by each vegetable?
Mixed Addition & Subtraction

Reasoning and Problem Solving

The mass of Annie's suitcase is $29 \frac{1}{2}$ kg.

Teddy's suitcase weighs $27 \frac{3}{10}$ kg. Teddy's suitcase is $2 \frac{1}{5}$ kg lighter than Annie's.

How much does Teddy's suitcase weigh? How much do the suitcases weigh altogether?

There is a weight allowance of 32 kg per suitcase.

How much below the weight allowance are Annie and Teddy?

Teddy's suitcase weighs $27 \frac{3}{10}$ kg. The suitcases weigh $56 \frac{4}{5}$ kg altogether.

Annie is $2 \frac{1}{2}$ kg under the weight allowance.

Teddy is $4 \frac{7}{10}$ kg under the weight allowance.

Find the value of the $\heartsuit + 3 \frac{4}{9} = 6 \frac{1}{3}$

$8 \frac{1}{10} - \heartsuit = \odot$

The value of the $\heartsuit$ is $2 \frac{8}{9}$.

The value of the $\odot$ is $5 \frac{19}{90}$. 

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Multiply by an Integer (1)

Notes and Guidance

Children are introduced to multiplying fractions by a whole number for the first time. They link this to repeated addition and see that the denominator remains the same, whilst the numerator is multiplied by the integer. This is shown clearly through the range of models to build the children’s conceptual understanding of multiplying fractions. Children should be encouraged to simplify fractions where possible.

Mathematical Talk

How is multiplying fractions similar to adding fractions?

What is the same/different between $\frac{3}{4} \times 2$ and $2 \times \frac{3}{4}$?

Which bar model do you find the most useful?

Which bar model helps us to convert from an improper fraction to a mixed number most effectively?

What has happened to the numerator/denominator?

Varied Fluency

Work out $\frac{1}{6} \times 4$ by counting in sixths.

$\frac{1}{6} \times 4 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

Use this method to work out:

$2 \times \frac{1}{3}$ $\quad \frac{1}{5} \times 3$ $\quad 6 \times \frac{1}{10}$

Mo uses a single bar model to work out: $\frac{1}{5} \times 4 = \frac{4}{5}$

Use this method to work out:

$\frac{1}{4} \times 3$ $\quad 6 \times \frac{1}{8}$ $\quad \frac{1}{10} \times 8$

Eva uses a number line and repeated addition to work out

$\frac{1}{5} \times 7 = \frac{7}{5} = 1 \frac{2}{5}$

Use this method to work out:

$5 \times \frac{1}{8}$ $\quad \frac{1}{3} \times 3$ $\quad \frac{1}{4} \times 7$
**Multiply by an Integer (1)**

**Reasoning and Problem Solving**

Amir is multiplying fractions by a whole number.

\[ \frac{1}{5} \times 5 = \frac{5}{25} \]

Can you explain his mistake?

Always, sometimes, never?

When you multiply a unit fraction by the same number as it’s denominator the answer will be one whole.

Always - because the numerator was 1 it will always be the same as your denominator when multiplied which means that it is a whole.

\[ \frac{1}{3} \times 3 = \frac{3}{3} = 1 \]

I am thinking of a unit fraction.

When I multiply it by 4 it will be equivalent to \( \frac{1}{2} \)

When I multiply it by 2 it will be equivalent to \( \frac{1}{4} \)

What is my fraction?

What do I need to multiply my fraction by so that my answer is equivalent to \( \frac{3}{4} \)?

Can you create your own version of this problem?

\[ \frac{1}{8} \text{ because} \]

\[ 4 \times \frac{1}{8} = \frac{4}{8} = \frac{1}{2} \]

\[ 2 \times \frac{1}{8} = \frac{2}{8} = \frac{1}{4} \]

\[ 6 \text{ because} \]

\[ 6 \times \frac{1}{8} = \frac{6}{8} = \frac{3}{4} \]
Multiplying by an Integer (2)

**Notes and Guidance**

Children apply prior knowledge of multiplying a unit fraction by a whole number to multiplying a non-unit fraction by a whole number.

They use similar models and discuss which method will be the most efficient depending on the questions asked.

Reinforce the concept of commutativity by showing examples of the fraction first and the integer first in the multiplication.

**Mathematical Talk**

Can you show me 3 lots of $\frac{3}{10}$ on a bar model?

How many tenths do we have altogether?

How does repeated addition help us with this multiplication?

How does a number line help us see the multiplication?
Multiply by an Integer (2)

Reasoning and Problem Solving

Use the digit cards only once to complete these multiplications.

\[ \begin{align*}
9 & \times \quad \frac{2}{3} = \quad \frac{9}{6} \\
1 & \times \quad \frac{2}{3} = \quad \frac{3}{6}
\end{align*} \]

Whitney has calculated \( 4 \times \frac{3}{14} \).

\[ \begin{align*}
2 & \times \quad \frac{3}{4} = \quad \frac{9}{6} \\
2 & \times \quad \frac{1}{3} = \quad \frac{4}{6} \\
2 & \times \quad \frac{1}{4} = \quad \frac{3}{6}
\end{align*} \]

Possible answer:

I disagree. Whitney has shaded 12 fourteenths. She has counted all of the boxes to give her the denominator when it is not needed. The answer should be \( \frac{12}{14} \) or \( \frac{6}{7} \).

Do you agree?

Explain why.
Multiply by an Integer (3)

Notes and Guidance

Children use their knowledge of fractions to multiply a mixed number by a whole number.

They use the method of repeated addition, multiplying the whole and part separately and the method of converting to an improper fraction then multiplying.

Continue to explore visual representations such as the bar model.

Mathematical Talk

How could you represent this mixed number?

What is the denominator? How do you know?

How many wholes are there? How many parts are there?

What is multiplying fractions similar to? (repeated addition)

What representation could you use to convert a mixed number to an improper fraction?

Varied Fluency

Use repeated addition to work out \(2 \frac{2}{3} \times 4\)

\[
2 \frac{2}{3} \times 4 = 2 \frac{2}{3} + 2 \frac{2}{3} + 2 \frac{2}{3} + 2 \frac{2}{3} = 8 \frac{8}{3} = 10 \frac{2}{3}
\]

Use this method to solve:

\[
2 \frac{1}{6} \times 3 \quad 1 \frac{3}{7} \times 2 \quad 3 \frac{1}{3} \times 4
\]

Partition your fraction to help you solve \(2 \frac{3}{4} \times 3\)

\[
\begin{align*}
2 \times 3 &= 6 \\
\frac{3}{4} \times 3 &= \frac{9}{4} = 2 \frac{1}{4} \\
6 + 2 \frac{1}{4} &= 8 \frac{1}{4}
\end{align*}
\]

Use this method to answer:

\[
2 \frac{5}{6} \times 3 \quad 3 \frac{4}{7} \times 2 \quad 2 \frac{1}{3} \times 5
\]

Convert to an improper fraction to calculate:

\[
3 \frac{2}{7} \times 4 \quad 2 \frac{4}{9} \times 2 \quad 4 \times 3 \frac{3}{5}
\]
### Multiply by an Integer (3)

#### Reasoning and Problem Solving

Jack runs $\frac{2}{3}$ miles three times per week.

Dexter runs $\frac{3}{4}$ miles twice a week.

Who runs the furthest during the week?

Explain your answer.

Jack runs $2 \frac{2}{3} \times 3 = 8$ miles.

Dexter runs $\frac{3}{4} \times 2 = 7 \frac{1}{2}$ miles.

Jack runs further by half a mile.

---

Work out the missing numbers.

Possible answer: $2 \frac{5}{8} \times 3 = 7 \frac{7}{8}$

I knew that the multiplier could not be 4 because that would give an answer of at least 8. So the multiplier had to be 3. That meant that the missing numerator had to give a product of 15. I knew that 5 multiplied by 3 would give 15.
Multiply Fractions by Integers

Notes and Guidance

Children multiply fractions and mixed numbers by integers. They use diagrams to highlight the link between multiplication and repeated addition. This supports the children in understanding why the denominator stays the same and we multiply the numerator.

When multiplying mixed numbers, children partition into wholes and parts to multiply more efficiently. They compare this method with multiplying improper fractions.

Mathematical Talk

How is multiplying fractions similar to adding fractions?

How does partitioning the mixed number into wholes and fractions support us to multiply?

Do you prefer partitioning the mixed number or converting it to an improper fraction to multiply? Why?

Does it matter if the integer is first or second in the multiplication sentence? Why?

Varied Fluency

- Complete:
  - \(3 \times \frac{2}{3}\)
  - \(4 \times \frac{7}{8}\)
  - \(\frac{2}{5} \times 7\)

- Eva partitions \(2\frac{3}{5}\) to help her to calculate \(2\frac{3}{5} \times 3\)

  \(2 \times 3 = 6\)
  \(\frac{3}{5} \times 3 = \frac{9}{5} = 1\frac{4}{5}\)
  \(6 + 1\frac{4}{5} = 7\frac{4}{5}\)

  Use Eva’s method to calculate:
  
  \(2\frac{5}{6} \times 3\)  \(1\frac{3}{7} \times 5\)  \(2\frac{2}{3} \times 3\)  \(4 \times 1\frac{1}{6}\)

- Convert the mixed number to an improper fraction to multiply.

  \(2\frac{3}{5} \times 3 = \frac{13}{5} \times 3 = \frac{39}{5} = 7\frac{4}{5}\)

  Use this method to calculate:

  \(3 \times 2\frac{2}{5}\)  \(1\frac{5}{7} \times 3\)  \(2 \times 1\frac{3}{4}\)  \(2 \times 1\frac{1}{6}\)
There are 9 lamp posts on a road. There is \(4 \frac{3}{8}\) of a metre between each lamp post.

What is the distance between the first and last lamp post?

Use pattern blocks, if is equal to 1 whole, work out what fraction the other shapes represent. Use this to calculate the multiplications. Give your answers in their simplest form.

- \(\triangle \times 5 = \frac{5}{6}\)
- \(\square \times 5 = \frac{5}{3} = 1\frac{2}{3}\)
- \(\square \times 5 = \frac{5}{2} = 2\frac{1}{2}\)

\[8 \times 4 \frac{3}{8} = 8 \times \frac{35}{8} = \frac{280}{8} = 35\]

The distance between the first and last lamp post is 35 metres.

Eva and Amir both work on a homework project.

- Eva spent \(4 \frac{1}{4}\) hours a week for 4 weeks doing my project.
- Amir spent \(2 \frac{3}{4}\) hours a week for 5 weeks doing my project.

Who spent the most time on their project?

Explain your reasoning.
Multiply Fractions by Fractions

Notes and Guidance

Children use concrete and pictorial representations to support them to multiply fractions. Support children in understanding the link between multiplying fractions and finding fractions of an amount: \( \frac{1}{3} \times \frac{1}{2} \) is the same as \( \frac{1}{3} \) of \( \frac{1}{2} \).
Encourage children to spot the patterns of what is happening in the multiplication, to support them in unpicking the procedure of multiplying fractions by multiplying the numerators and multiplying the denominators.

Mathematical Talk

Could you use folding paper to calculate \( \frac{2}{3} \times \frac{1}{2} \)? How? Use a piece of paper to model this to a friend.

How are the diagrams similar to folding paper? Which do you find more efficient?

What do you notice about the product of the fractions you have multiplied? What is the procedure to multiply fractions?

Does multiplying two numbers always give you a larger product? Explain why.

Varied Fluency

Dexter is calculating \( \frac{1}{3} \times \frac{1}{2} \) by folding paper. He folds a piece of paper in half. He then folds the half into thirds. He shades the fraction of paper he has created. When he opens it up he finds he has shaded \( \frac{1}{6} \) of the whole piece of paper.

\( \frac{1}{3} \times \frac{1}{2} \) means \( \frac{1}{3} \) of a half. Folding half the paper into three equal parts showed me that \( \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \).

Represent and calculate the multiplications by folding paper.

\[
\frac{1}{4} \times \frac{1}{2} = \quad \frac{1}{4} \times \frac{1}{3} = \quad \frac{1}{4} \times \frac{1}{4} =
\]

Alex is drawing diagrams to represent multiplying fractions.

Shade the diagrams to calculate:

\[
\frac{1}{3} \times \frac{1}{2} = \quad \frac{1}{4} \times \frac{1}{2} = \quad \frac{1}{3} \times \frac{1}{4} = \quad \frac{2}{3} \times \frac{1}{4} = \quad \frac{2}{3} \times \frac{3}{4} =
\]

Write your answers in their simplest form.
Multiply Fractions by Fractions

Reasoning and Problem Solving

The shaded square in the grid below is the answer to a multiplying fractions question. What was the question?

How many ways can you complete the missing digits?

Possible answers:
\[ \frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2} \]

Children could also use improper fractions.

Find the area of the shaded part of the shape.

Not drawn accurately

\[ \frac{1}{4} \times \frac{1}{2} \]

is the same as \(\frac{1}{2}\) of a quarter.

Do you agree? Explain why.

Alex says,

\[ \frac{1}{4} \times \frac{1}{2} \]

is the same as \(\frac{1}{2}\) of a quarter.

Alex is correct. Multiplication is commutative so \(\frac{1}{4} \times \frac{1}{2}\) is the same as \(\frac{1}{2}\) of a quarter or \(\frac{1}{4}\) of a half.

The shaded area is \(\frac{11}{21}\) m\(^2\).
**Divide Fractions by Integers (1)**

### Notes and Guidance

Children are introduced to dividing fractions by integers for the first time. They focus on dividing fractions where the numerator is a multiple of the integer they are dividing by. Encourage children to spot the pattern that the denominator stays the same and the numerator is divided by the integer.

Children link dividing fractions to multiplying by unit fractions. Use the diagrams children drew for multiplying fractions to discuss how and why the calculations are similar.

### Mathematical Talk

- **How could you represent this fraction?**
- **Is the numerator divisible by the integer?**
- **Why doesn’t the denominator change?**

- **What pattern can you see when dividing elevenths?**
- **How can we use the pattern to help us to calculate a mixed number by an integer? Can you convert it to an improper fraction?**

---

**Varied Fluency**

Dexter has \( \frac{2}{5} \) of a chocolate bar. He shares it with his friend. What fraction of the chocolate bar do they each get?

Use the diagrams to help you calculate.

\[ \frac{3}{4} \div 3 = \quad \frac{4}{7} \div 4 = \quad \frac{4}{7} \div 2 = \]

Calculate.

\[ \frac{1}{11} \div 1 = \quad \frac{2}{11} \div 2 = \quad \frac{3}{11} \div 3 = \quad \frac{4}{11} \div 4 = \]

\[ \frac{2}{11} \div 2 = \quad \frac{4}{11} \div 2 = \quad \frac{6}{11} \div 2 = \quad \frac{8}{11} \div 2 = \]

\[ \frac{3}{11} \div 3 = \quad \frac{6}{11} \div 3 = \quad \frac{9}{11} \div 3 = \quad 1 \frac{1}{11} \div 3 = \]
Tommy says,

Dividing by 2 is the same as finding half of a number so \( \frac{4}{11} \div 2 \) is the same as \( \frac{1}{2} \times \frac{4}{11} \)

Do you agree? Explain why.

Tommy is correct. It may help children to understand this by reinforcing that \( \frac{1}{2} \times \frac{4}{11} \) is the same as \( \frac{1}{2} \) of \( \frac{4}{11} \)

Complete the missing integers.

\[
\begin{align*}
\frac{15}{16} \div \Box &= \frac{5}{16} \\
\frac{15}{16} \div \Box &= \frac{3}{16} \\
\frac{20}{23} \div \Box &= \frac{4}{23} \\
\frac{20}{23} \div \Box &= \frac{5}{23}
\end{align*}
\]

Rosie walks for \( \frac{3}{4} \) of an hour over 3 days. She walks for the same amount of time each day. How many minutes does Rosie walk each day?

Rosie walks for \( \frac{1}{4} \) of an hour each day. She walks for 15 minutes each day.

Match the equivalent calculations.

\[
\begin{align*}
\frac{1}{4} \times \frac{12}{13} &= \frac{12}{13} \div 4 \\
\frac{1}{6} \times \frac{12}{13} &= \frac{12}{13} \div 6 \\
\frac{1}{2} \times \frac{12}{13} &= \frac{12}{13} \div 2 \\
\frac{1}{3} \times \frac{12}{13} &= \frac{12}{13} \div 3
\end{align*}
\]
Divide Fractions by Integers (2)

Notes and Guidance

Children divide fractions where the numerator is not a multiple of the integer they are dividing by. They draw diagrams to divide fractions into equal parts and explore the link between multiplying by a unit fraction and dividing by an integer. Children find equivalent fractions to support the divisions and draw diagrams to model how this works.

Mathematical Talk

How is Mo’s method of dividing fractions similar to multiplying $\frac{1}{3}$ by $\frac{1}{2}$?

Do you prefer Mo’s or Annie’s method? Explain why.

Why does finding an equivalent fraction help us to divide fractions by integers?

What multiplication can I use to calculate $\frac{3}{5} \div 2$? Explain how you know.

Varied Fluency

Mo is dividing $\frac{1}{3}$ by 2

I have divided one third into 2 equal parts. Each part is worth $\frac{1}{6}$.

$$\frac{1}{3} \div 2 = \frac{1}{6}$$

Draw diagrams to calculate:

$$\frac{1}{3} \div 3 = \frac{2}{3} \div 3 = \frac{1}{5} \div 3 = \frac{2}{5} \div 3 =$$

Annie is dividing $\frac{2}{3}$ by 4

The numerator isn’t a multiple of the integer I am dividing by so I will find an equivalent fraction to help me divide the numerator equally.

Find equivalent fractions to calculate:

$$\frac{3}{5} \div 2 = \frac{1}{3} \div 3 = \frac{2}{3} \div 3 =$$
Divide Fractions by Integers (2)

Reasoning and Problem Solving

Alex says,

I can only divide a fraction by an integer if the numerator is a multiple of the divisor.

Do you agree? Explain why.

Alex is wrong, we can divide any fraction by an integer.

Calculate the missing fractions and integers.

\[
\begin{align*}
\square & \div 4 = \frac{7}{36} \\
\frac{3}{20} & \div \square = \frac{3}{80} \\
\square & \div \square = \frac{2}{5}
\end{align*}
\]

Is there more than one possibility?

There are many possibilities in this last question. Children could look for patterns between the fractions and integers.
Four Rules with Fractions

Children combine the four operations when calculating with fractions. This is a good opportunity to recap the order of operations as children calculate equations with and without brackets. Encourage children to draw bar models to represent worded problems in order to understand which operation they need to use?

Mathematical Talk

Which part of the equation do we calculate first when we have more than one operation?

What do you notice about the six questions that begin with \(3 \frac{1}{3}\)?

What's the same about the equations? What's different?

Which equation has the largest answer? Can you order the answers to the equations in descending order?

Can you write the worded problem as a number sentence?

Varied Fluency

Complete the missing boxes.

\[ \begin{array}{c|c|c}
\frac{5}{8} & \frac{2}{5} & \\
\hline
\frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\
\hline
1\frac{1}{2} & & \\
\end{array} \]

Calculate:

\[3 \frac{1}{3} + \frac{1}{3} - 2 = \quad 3 \frac{1}{3} + \frac{1}{3} + 2 = \quad 3 \frac{1}{3} + \frac{1}{3} \times 2 =\]

\[3 \frac{1}{3} + \frac{1}{3} \div 2 = \quad (3 \frac{1}{3} + \frac{1}{3}) \times 2 = \quad (3 \frac{1}{3} + \frac{1}{3}) \div 2 = \]

Jack has one quarter of a bag of sweets and Whitney has two thirds of a bag of sweets. They combined their sweets and shared them equally between themselves and Rosie. What fraction of the sweets does each child receive?
### Four Rules with Fractions

#### Reasoning and Problem Solving

Add two sets of brackets to make the following calculation correct:

$$\frac{1}{2} + \frac{1}{4} \times 8 + \frac{1}{6} \div 3 = \frac{61}{18}$$

Explain where the brackets go and why. Did you find any difficulties?

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$\left(\frac{2}{3} + \frac{2}{9}\right) \div 4$$</td>
<td>$\frac{5}{9}$</td>
</tr>
<tr>
<td>$\frac{2}{3} - \frac{1}{3} \div 3$</td>
<td>$\frac{2}{9}$</td>
</tr>
<tr>
<td>$\frac{1}{3} \times 2 - \left(1\frac{1}{9} \div 2\right)$</td>
<td>$\frac{1}{9}$</td>
</tr>
</tbody>
</table>

Match each calculation to the correct answer.
Block 3 – Fractions

Theme 9 – Fraction of an amount
Fraction of an Amount

Notes and Guidance

Children recap previous learning surrounding finding unit and non-unit fractions of amounts, quantities and measures.

It is important that the concept is explored pictorially through bar models to support children to make sense of the abstract.

Varied Fluency

Find \( \frac{1}{7} \) of 42

\[ 42 \div 7 = 6 \]

\( \frac{1}{7} \) of 42 is 6

Use this method to find:

\( \frac{1}{8} \) of 56

\( \frac{1}{6} \) of 480

\( \frac{1}{9} \) of 81 m

Find \( \frac{2}{7} \) of 42

\[ 42 \div 7 = 6 \]

\( 6 \times 2 = 12 \)

\( \frac{2}{7} \) of 42 is 12

Use this method to find:

\( \frac{3}{8} \) of 56

\( \frac{5}{6} \) of 480

\( \frac{4}{9} \) of 81 m

Mathematical Talk

How many equal groups have you shared 49 into? Why?

What does each equal part represent as a fraction and an amount?

What could you do to 1 metre to make the calculation easier?

1 litre = ____ ml  
1 kg = ____ g

Draw a bar model to help you calculate:

\( \frac{4}{5} \) of 1 m

\( \frac{5}{12} \) of 1.44 litres

\( \frac{3}{7} \) of 21 kg
Fraction of an Amount

Reasoning and Problem Solving

Write a problem that matches the bar model.

Possible response:
There are 96 cars in a car park. 
\( \frac{3}{8} \) of them are red.
How many cars are red?
How many were not red? etc.

What other questions could you ask from this model?

Find the area of each colour in the rectangle.

Area of rectangle: 
\( 6 \times 8 = 48 \text{ cm}^2 \)
Blue 
\( \frac{4}{12} \) of 48 = 16 cm²
Red 
\( \frac{3}{12} \) of 48 = 12 cm²
Green 
\( \frac{5}{12} \) of 48 = 20 cm²

Children need to show that this would impact both the blue and the other colour.

\( \frac{7}{16} \) of a class are boys.

There are 32 children in the class.

How many children are in the class?

There are 18 girls in the class.
Fractions as Operators

Notes and Guidance
Children link their understanding of fractions of amounts and multiplying fractions to use fractions as operators.

They use their knowledge of commutativity to help them understand that you can change the order of multiplication without changing the product.

Varied Fluency
Tommy has calculated and drawn a bar model for two calculations.

\[ 5 \times \frac{3}{5} = \frac{15}{5} = 3 \]
\[ \frac{3}{5} \text{ of } 5 = 3 \]

What’s the same and what’s different about Tommy’s calculations?

Complete:
2 lots of \( \frac{1}{10} = \) [ ] \[ \frac{1}{10} \text{ of } 2 = [ ] \]
6 lots of [ ] = 3 [ ] of 6 = 3
8 lots of \( \frac{1}{4} = [ ] \) \[ \frac{1}{4} \text{ of } 8 = [ ] \]

Use this to complete:
\[ 20 \times \frac{4}{5} = [ ] \text{ of } 20 = [ ] \]
\[ [ ] \times \frac{2}{3} = [ ] \text{ of } 18 = 12 \]
\[ [ ] \times \frac{1}{3} = \frac{1}{3} \text{ of } [ ] = 20 \]

Mathematical Talk
What is the same and different about these bar models?

Is it easier to multiply a fraction or find a fraction of an amount? Does it depend on the whole number you are multiplying by? Can you see the link between the numbers?
Fractions as Operators

Reasoning and Problem Solving

Which method would you use to complete these calculations: multiply the fractions or find the fraction of an amount?

Explain your choice for each one. Compare your method to your partner.

Possible response:
1. Children may find it easier to find 3 fifths of 25 rather than multiply 25 by 3
2. Children may choose either as they are of similar efficiency.
3. Children will probably find it more efficient to multiply than divide 5 by 8

Dexter and Jack are thinking of a two-digit number between 20 and 30

Dexter finds two thirds of the number.

Jack multiplies the number by $\frac{2}{3}$

Their new two-digit number has a digit total that is one more than that of their original number.

What number did they start with?

Show each step of their calculation.

They started with 24

Dexter:
$24 \div 3 = 8$
$8 \times 2 = 16$

Jack:
$24 \times 2 = 48$
$48 \div 3 = 16$
Fraction of an Amount

Notes and Guidance

Children calculate fractions of an amount. They recognise that the denominator is the number of parts the amount is being divided into, and the numerator is the amount of those parts we need to know about.
Encourage children to draw bar models to support the procedure of dividing by the denominator and multiplying by the numerator to find fractions of amounts.

Varied Fluency

A cook has 48 kg of potatoes. He uses \( \frac{5}{8} \) of the potatoes. How many kilograms of the potatoes does he have left?
Use the bar model to find the answer to this question.

A football team has 300 tickets to give away. They give \( \frac{3}{4} \) of them to a local school. How many tickets are left?

Mathematical Talk

What is the value of the whole?
How many equal parts are there altogether?
How many equal parts do we need?
What is the value of each equal part?
Can you see a pattern in the questions starting with \( \frac{1}{5} \) of 30?
What would the next column to the right of the questions be?
What would the next row of questions underneath be? How do you know? How can you predict the answers?

Calculate:
\[ \frac{1}{5} \text{ of } 30 = \quad \frac{1}{5} \text{ of } 60 = \quad \frac{1}{5} \text{ of } 120 = \quad \frac{1}{5} \text{ of } 240 = \]
\[ \frac{2}{5} \text{ of } 30 = \quad \frac{1}{5} \text{ of } 600 = \quad \frac{1}{10} \text{ of } 120 = \quad \frac{6}{5} \text{ of } 240 = \]
\[ \frac{4}{5} \text{ of } 30 = \quad \frac{1}{5} \text{ of } 6,000 = \quad \frac{1}{20} \text{ of } 120 = \quad \frac{11}{5} \text{ of } 240 = \]
Fraction of an Amount

Reasoning and Problem Solving

What is the value of A?
What is the value of B?

A = 648
B = 540

Two fashion designers receive $\frac{3}{8}$ of 208 metres of material.

One of them says:
We each receive 26 m

Is she correct?
Explain your reasoning.

Calculate the missing digits.

$\frac{3}{8}$ of 40 = $\frac{?}{10}$ of 150

$\frac{1}{5}$ of 315 = $\frac{?}{8}$ of 72

She is incorrect because 26 is only one eighth of 208. She needs to multiply her answer by 3 so that they each get 78 m each.
Find the Whole

Notes and Guidance

Children find the whole amount from the known value of a fraction. Encourage children to continue to use bar models to support them in representing the parts and the whole. Children will consider looking for patterns when calculating the whole. Highlight the importance of multiplication and division when calculating fractions of amounts and how knowing our times-tables can support us to calculate the whole more efficiently.

Mathematical Talk

How many equal parts are there altogether?
How many equal parts do we know?
What is the value of each equal part?
What is the value of the whole?
Can you see a pattern in the questions?
How can we find the whole?
Can you estimate what the answer is? Can you check the answer using a bar model?

Varied Fluency

Jack has spent \( \frac{2}{3} \) of his money.
He spent £60, how much did he have to start with?

£60

Use a bar model to represent and solve the problems.

- Rosie eats \( \frac{2}{5} \) of a packet of biscuits. She eats 10 biscuits. How many biscuits were in the original packet?
- In an election, \( \frac{3}{8} \) of a town voted. If 120 people voted, how many people lived in the town?

Calculate:

\[ \frac{1}{4} \text{ of } \_ = 12 \quad \frac{1}{4} \text{ of } \_ = 36 \quad \frac{1}{4} \text{ of } \_ = 108 \]

\[ \frac{1}{12} \text{ of } \_ = 12 \quad \frac{3}{4} \text{ of } \_ = 36 \quad \frac{4}{4} \text{ of } \_ = 108 \]
Eva lit a candle while she had a bath. After her bath, \(\frac{2}{5}\) of the candle was left. It measured 13 cm. Eva says:

Is she correct?

Explain your reasoning.

Before my bath the candle measured 33 cm

She is incorrect. 13 ÷ 2 = 6.5

6.5 × 5 = 32.5 cm

She either didn’t halve correctly or didn’t multiply correctly

Rosie and Jack are making juice. They use \(\frac{6}{7}\) of the water in a jug and are left with this amount of water:

To work out how much we had originally, we should divide 300 by 6 then multiply by 7

No, we know that 300ml is \(\frac{1}{7}\) so we need to multiply it by 7

Who is correct?

Explain your reasoning.

Many possibilities. \(\frac{5}{8}\) of children have blue eyes. 15 children do not have blue eyes. How many children are there altogether?

Rosie is correct. Jack would only be correct if \(\frac{6}{7}\) was remaining but \(\frac{6}{7}\) is what was used. Rosie recognised that \(\frac{1}{7}\) is left in the jug therefore multiplied it by 7 to correctly find the whole.