Autumn Scheme of Learning

Year 5/6

#MathsEveryoneCan

2019-20
How to use the mixed-age SOL

In this document, you will find suggestions of how you may structure a progression in learning for a mixed-age class.

Firstly, we have created a yearly overview.

<table>
<thead>
<tr>
<th>Autumn</th>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
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</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>Week 2</td>
<td>Week 3</td>
<td>Week 4</td>
</tr>
<tr>
<td>Number: Place Value</td>
<td>Number: Addition and Subtraction</td>
<td>Number: Place Value to 50 and Multiplication</td>
<td>Number: Year 1: Division &amp; Consolidation Year 2: Division</td>
</tr>
<tr>
<td>Y1: Numbers to 20</td>
<td>Year 1: Numbers within 20 (including recognising money)</td>
<td>Year 1: Place Value to 50 and Multiplication</td>
<td>Year 1: Place Value to 100</td>
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<tr>
<td>Week 5</td>
<td>Week 6</td>
<td>Week 7</td>
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<tr>
<td>Geometry: Year 1: Shape and Consolidation</td>
<td>Year 2: Properties of Shape</td>
<td>Year 1: Fractions and Consolidation</td>
<td>Year 2: Fractions</td>
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<td>Week 9</td>
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For each block of learning, we have grouped the small steps into themes that have similar content. Within these themes, we list the corresponding small steps from one or both year groups. Teachers can then use the single-age schemes to access the guidance on each small step listed within each theme.

The themes are organised into common content (above the line) and year specific content (below the line). Moving from left to right, the arrows on the line suggest the order to teach the themes.

Each term has 12 weeks of learning. We are aware that some terms are longer and shorter than others, so teachers may adapt the overview to fit their term dates.

The overview shows how the content has been matched up over the year to support teachers in teaching similar concepts to both year groups. Where this is not possible, it is clearly indicated on the overview with 2 separate blocks.
Notes and Guidance

How to use the mixed-age SOL

Here is an example of one of the themes from the Year 1/2 mixed-age guidance.

<table>
<thead>
<tr>
<th>Subtraction</th>
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</thead>
<tbody>
<tr>
<td>Year 1 (Aut B2, Spr B1)</td>
</tr>
<tr>
<td>• How many left? (1)</td>
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<td>• How many left? (2)</td>
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<tr>
<td>• Counting back</td>
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<td>• Subtraction - not crossing 10</td>
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<tr>
<td>• Subtraction - crossing 10 (1)</td>
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<td>• Subtraction - crossing 10 (2)</td>
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<tr>
<td>Year 2 (Aut B2, B3)</td>
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<tr>
<td>• Subtract 1-digit from 2-digits</td>
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<tr>
<td>• Subtract with 2-digits (1)</td>
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<td>• Subtract with 2-digits (2)</td>
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<tr>
<td>• Find change - money</td>
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</table>

In order to create a more coherent journey for mixed-age classes, we have re-ordered some of the single-age steps and combined some blocks of learning e.g. Money is covered within Addition and Subtraction.

The bullet points are the names of the small steps from the single-age SOL. We have referenced where the steps are from at the top of each theme e.g. Aut B2 means Autumn term, Block 2. Teachers will need to access both of the single-age SOLs from our website together with this mixed-age guidance in order to plan their learning.

Points to consider

• Use the mixed-age schemes to see where similar skills from both year groups can be taught together. Learning can then be differentiated through the questions on the single-age small steps so both year groups are focusing on their year group content.
• When there is year group specific content, consider teaching in split inputs to classes. This will depend on support in class and may need to be done through focus groups.
• On each of the block overview pages, we have described the key learning in each block and have given suggestions as to how the themes could be approached for each year group.
• We are fully aware that every class is different and the logistics of mixed-age classes can be tricky. We hope that our mixed-age SOL can help teachers to start to draw learning together.
Guidance

Common Content

In this section, content from single-age blocks are matched together to show teachers where there are clear links across the year groups. Teachers may decide to teach the lower year’s content to the whole class before moving the higher year on to their age-related expectations. The lower year group is not expected to cover the higher year group’s content as they should focus on their own age-related expectations.

Year Specific

In this section, content that is discrete to one year group is outlined. Teachers may need to consider a split input with lessons or working with children in focus groups to ensure they have full coverage of their year’s curriculum. Guidance is given on each page to support the planning of each block.

The themes should be taught in order from left to right.
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<th>Week 1</th>
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<td>Number: Decimals and Percentages</td>
<td>Y5: Number: Decimals</td>
<td>Y5: Consolidation</td>
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<td>Measurement: Converting Units</td>
<td>Measurement: Perimeter, Area and Volume</td>
<td>Y6: Number: Ratio</td>
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<td>Geometry: Position and Direction</td>
<td>Y6: SATS</td>
<td>Investigations and Consolidation</td>
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Four Operations (1)

Common Content

**Addition and subtraction**
- Year 5 (Aut B2)
  - Add whole numbers with more than 4-digits
  - Subtract whole numbers with more than 4-digits
  - Inverse operations
  - Multi-step addition and subtraction problems
- Year 6 (Aut B2)
  - Add and subtract whole numbers

**Multiples**
- Year 5 (Aut B4)
  - Multiples
- Year 6 (Aut B2)
  - Common multiples

**Multiplication**
- Year 5 (Spr B1)
  - Multiply 4-digits by 1-digit
  - Multiply 2-digits (area model)
  - Multiply 2-digits by 2-digits
  - Multiply 3-digits by 2-digits
  - Multiply 4-digits by 2-digits
- Year 6 (Aut B2)
  - Multiply 4-digits by 2-digits

**Factors**
- Year 5 (Aut B4)
  - Factors
  - Common factors
- Year 6 (Aut B2)
  - Common factors

In this block, Year 6 have a lot of opportunities to recap prior learning as Year 5 are introduced to content for the first time.

Building on previous year groups, children add and subtract larger numbers and use their skills to solve problems.

Children then focus on multiplication. Year 5 break down their learning into 5 small steps however Year 6 could also use this opportunity to build their skills towards their final aim of multiplying up to 4-digits by 1 or 2-digit numbers.
### Four Operations (2)

#### Common Content

**Division**
- Year 5 (Spr B1)
  - Divide 4-digits by 1-digit
  - Divide with remainders
- Year 6 (Aut B2)
  - Short division
  - Division using factors
  - Long division (1)
  - Long division (2)
  - Long division (3)
  - Long division (4)

**Primes, Squares and Cubes**
- Year 5 (Aut B4)
  - Prime numbers
  - Square numbers
  - Cube numbers
- Year 6 (Aut B2)
  - Primes
  - Squares and Cubes

**Estimating**
- Year 5 (Aut B2)
  - Round to estimate and approximate
- Year 6 (Aut B2)
  - Mental calculations and estimation

Both year groups divide numbers using short division including remainders. Year 6 then move on to look at long division.

Drawing learning together, Year 6 look at order of operations and reasoning from known facts whilst Year 5 focus on fluency within the four operations.

### Year Specific

**Order of operations**
- Year 6 (Aut B2)
  - Order of Operations

**Related facts**
- Year 6 (Aut B2)
  - Reason from known facts
Four operations

Theme 1 – Addition and subtraction
Add More than 4-digits

Notes and Guidance

Children will build upon previous learning of column addition. They will now look at numbers with more than four digits and use their place value knowledge to line the numbers up accurately.

Children use a range of manipulatives to demonstrate their understanding and use pictorial representations to support their problem solving.

Mathematical Talk

Will you have to exchange? How do you know which columns will be affected?

Does it matter that the two numbers don’t have the same amount of digits?

Which number goes on top in the calculation? Does it affect the answer?

Varied Fluency

Ron uses place value counters to calculate 4,356 + 2,435

Use Ron’s method to calculate:

Jack, Rosie and Eva are playing a computer game. Jack has 3,452 points, Rosie has 4,039 points and Eva has 10,989 points.

How many points do Jack and Rosie have altogether?
How many points do Rosie and Eva have altogether?
How many points do Jack and Eva have altogether?
How many points do Jack, Rosie and Eva have altogether?
Add More than 4-digits

Reasoning and Problem Solving

Amir is discovering numbers on a Gattegno chart.

He makes this number.

He moved the counter on the thousands row, he moved it from 4,000 to 7,000.

Amir moves one counter three spaces on a horizontal line to create a new number.

When he adds this to his original number he gets 131,130.

Which counter did he move?

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Work out the missing numbers.

54,937 + 23,592 = 78,529

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</table>
Subtract More than 4-digits

Notes and Guidance

Building on Year 4 experience, children use their knowledge of subtracting using the formal column method to subtract numbers with more than four digits. Children will be focusing on exchange and will be concentrating on the correct place value. It is important that children know when an exchange is and isn’t needed. Children need to experience ‘0’ as a place holder.

Mathematical Talk

Why is it important that we start subtracting the smallest place value first?

Does it matter which number goes on top? Why? Will you have to exchange? How do you know which columns will be affected?

Does it matter that the two numbers don’t have the same amount of digits?

Varied Fluency

Calculate:

4,648 − 2,347

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<tr>
<th>1,000s</th>
<th>100s</th>
<th>10s</th>
<th>1s</th>
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45,536 − 8,426

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<td>8</td>
<td>4</td>
<td>2</td>
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</table>

Represent each problem as a bar model, and solve them.

A plane is flying at 29,456 feet. During the flight the plane descends 8,896 feet. What height is the plane now flying at?

Tommy earns £37,506 pounds a year. Dora earns £22,819 a year. How much more money does Tommy earn than Dora?

There are 83,065 fans at a football match. 45,927 fans are male. How many fans are female?
Subtract More than 4-digits

Reasoning and Problem Solving

Eva makes a 5-digit number.

Mo makes a 4-digit number.

The difference between their numbers is 3,465

What could their numbers be?

Possible answers:
9,658 and 14,023
12,654 and 8,289
5,635 and 10,000
Etc.

Rosie completes this subtraction incorrectly.

```
28701
- 7621
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21180
```

Explain the mistake to Rosie and correct it for her.

Rosie did not write down the exchange she made when she exchanged 1 hundred for 10 tens. This means she still had 7 hundreds subtract 6 hundreds when she should have 6 hundreds subtract 6 hundreds. The correct answer is 21,080.
Inverse Operations

Notes and Guidance

In this small step, children will use their knowledge of addition and subtraction to check their workings to ensure accuracy.

They use the commutative law to see that addition can be done in any order but subtraction cannot.

Varied Fluency

- When calculating 17,468 – 8,947, which answer gives the corresponding addition question?

  - 8,947 + 8,631 = 17,468
  - 8,947 + 8,521 = 17,468
  - 8,251 + 8,947 = 17,468

- I’m thinking of a number. After I add 5,241 and subtract 352, my number is 9,485. What was my original number?

- Eva and Dexter are playing a computer game. Eva’s high score is 8,524. Dexter’s high score is greater than Eva’s. The total of both of their scores is 19,384. What is Dexter’s high score?

Mathematical Talk

- How can you tell if your answer is sensible?

- What is the inverse of addition?

- What is the inverse of subtraction?
Inverse Operations

Reasoning and Problem Solving

Complete the pyramid using addition and subtraction.

From left to right:
- Bottom row: 3,804, 5,005
- Second row: 8,118
- Third row: 15,094, 13,391
- Fourth row: 28,485, 27,422

Mo, Whitney, Teddy and Eva collect marbles.

Eva has 2,756 marbles.

Mo has 1,648 marbles.

Whitney has double the amount of marbles Mo has.

Teddy has half the amount of marbles Mo has.

In total they have 8,524 marbles between them. How many does Eva have?
Multi-step Problems

Notes and Guidance

In this small step children will be using their knowledge of addition and subtraction to solve multi-step problems.

The problems will appear in different contexts and in different forms i.e. bar models and word problems.

Varied Fluency

When Annie opened her book, she saw two numbered pages. The sum of these two pages was 317. What would the next page number be?

Adam is twice as old as Barry. Charlie is 3 years younger than Barry. The sum of all their ages is 53. How old is Barry?

The sum of two numbers is 11,339. The difference between the same two numbers is 1,209. Use the bar model to help you find the numbers.

Mathematical Talk

What is the key vocabulary in the question?

What are the key bits of information?

Can we put this information into a model?

Which operations do we need to use?
Multi-step Problems

Reasoning and Problem Solving

A milkman has 250 bottles of milk.
He collects another 160 from the dairy, and delivers 375 during the day.
How many does he have left?

My method:
375 – 250 = 125
125 + 160 = 285

Tommy is wrong. He should have added 250 and 160, then subtracted 375 from the answer.

There are 35 bottles of milk remaining.

Tommy

Do you agree with Tommy? Explain why.

On Monday, Whitney was paid £114

On Tuesday, she was paid £27 more than on Monday.

On Wednesday, she was paid £27 less than on Monday.

How much was Whitney paid in total?

How many calculations did you do?

Is there a more efficient method?

£342

Children might add 114 and 27, subtract 27 from 114 and then add their numbers.

A more efficient method is to recognise that the ‘£27 more’ and ‘£27 less’ cancel out so they can just multiply £114 by three.
Add & Subtract Integers

Notes and Guidance

Children consolidate their knowledge of column addition and subtraction, reinforcing the language of ‘exchange’ etc. After showing confidence with smaller numbers, children should progress to multi-digit calculations. Children will consider whether the column method is always appropriate e.g. when adding 999, it is easier to add 1,000 then subtract 1
They use these skills to solve multi-step problems in a range of contexts.

Mathematical Talk

What happens when there is more than 9 in a place value column?
Can you make an exchange between columns?
How can we find the missing digits? Can we use the inverse?
Is the column method always the best method?
When should we use mental methods?

Varied Fluency

Calculate.

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67,832 + 5,258
834,501 – 299,999

A four bedroom house costs £450,000
A three bedroom house costs £201,000 less.
How much does the three bedroom house cost?
What method did you use to find the answer?

Find the missing digits. What do you notice?

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9 | 0 | ? | 3 | ? | 2
Add & Subtract Integers

Reasoning and Problem Solving

Find the difference between A and B.

A = 19,000
B = 50,500
The difference is 31,500

Here is a bar model.

A = 99,255
B = 532,000

Possible answer:

A is an odd number which rounds to 100,000 to the nearest ten thousand. It has a digit total of 30.

B is an even number which rounds to 500,000 to the nearest hundred thousand. It has a digit total of 10.

A and B are multiples of 5.

What are possible values of A and B?
### Multiples

#### Notes and Guidance

Building on their times tables knowledge, children will find multiples of whole numbers. Children build multiples of a number using concrete and pictorial representation. e.g. an array. Children understand that a multiple of a number is the product of the number and another whole number.

Multiplying decimal numbers by 10, 100 and 1,000 forms part of Year 5 Summer block 1.

#### Mathematical Talk

What do you notice about the multiples of 5? What is the same about each of them, what is different?

Look at multiples of other numbers, is there a pattern that links them to each other?

Are all multiples of 8 multiples of 4?

Are all multiples of 4 multiples of 8?

### Varied Fluency

#### Circle the multiples of 5

- 25
- 32
- 54
- 175
- 554
- 3000

What do you notice about the multiples of 5?

#### 7,135 is a multiple of 5. Explain how you know.

#### Roll 2 dice (1-6), and multiply the numbers you roll. List all the numbers that this number is a multiple of. Repeat the dice roll. Use a table to show your results. Multiply the numbers you roll to complete the table.
Multiples

Reasoning and Problem Solving

Use 0 – 9 digit cards. Choose 2 cards and multiply the digits shown.

What is your number a multiple of?

Is it a multiple of more than one number?

Find all the numbers you can make using the digit cards.

Use the table below to help.

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</tbody>
</table>

Always, Sometimes, Never

- The product of two even numbers is a multiple of an odd number.
- The product of two odd numbers is a multiple of an even number.

Eva’s age is a multiple of 7 and is 3 less than a multiple of 8
She is younger than 40
How old is Eva?

Always - all integers are multiples of 1, which is an odd number.

Never - Two odd numbers multiplied together are always a multiple of an odd number.

Eva is 21 years old.
**Common Multiples**

**Notes and Guidance**

Building on knowledge of multiples, children find common multiples of numbers. They should continue to use visual representations to support their thinking.

They also use abstract methods to calculate multiples, including using numbers outside of those known in times table facts.

**Mathematical Talk**

Is the lowest common multiple of a pair of numbers always the product of them?

Can you think of any strategies to work out the lowest common multiples of different numbers?

When do numbers have common multiples that are lower than their product?

---

**Varied Fluency**

On a 100 square, shade the first 5 multiples of 7 and then the first 8 multiples of 5.

What common multiple of 7 and 5 do you find?

Use this number to find other common multiples of 7 and 5.

List 5 common multiples of 4 and 3.

Alex and Eva play football at the same local football pitches. Alex plays every 4 days and Eva plays every 6 days.

They both played football today.

After a fortnight, how many times will they have played football on the same day?
Common Multiples

Reasoning and Problem Solving

Work out the headings for the Venn diagram.

Multiply of 4
Multiply of 6

144 is a square number that can go in the middle.

Annie is double her sister's age.

They are both older than 20 but younger than 50

Their ages are both multiples of 7

What are their ages?

A train starts running from Leeds to York at 7am.
The last train leaves at midnight.

Platform 1 has a train leaving from it every 12 minutes.
Platform 2 has one leaving from it every 5 minutes.

How many times in the day would there be a train leaving from both platforms at the same time?

Annie is 42 and her sister is 21

Add in one more number to each section.

Can you find a square number that will go in the middle section of the Venn diagram?

18 times
Four operations

Theme 3 – Multiply and divide by multiples of 10
Multiply by 10, 100 and 1,000

Notes and Guidance

Children recap multiplying by 10 and 100 before moving on to multiplying by 1,000
They look at numbers in a place value grid and discuss the number of places to the left digits move when you multiply by different multiples of 10

Mathematical Talk

Which direction do the digits move when you multiply by 10, 100 or 1,000?
How many places do you move to the left?
When we have an empty place value column to the right of our digits what number do we use as a place holder?
Can you use multiplying by 100 to help you multiply by 1,000? Explain why.

Varied Fluency

Make 234 on a place value grid using counters.

<table>
<thead>
<tr>
<th>HTh</th>
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</thead>
<tbody>
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</tbody>
</table>

When I multiply 234 by 10, where will I move my counters?
Is this always the case when multiplying by 10?

Complete the following questions using counters and a place value grid.

234 \times 100 = ___
100 \times 36 = ___
45,020 \times 10 = ___

324 \times 100 = ___
1,000 \times 207 = ___
3,406 \times 1,000 = ___

Use <, > or = to complete the statements.

71 \times 1,000 ___ 71 \times 100
100 \times 32 ___ 16 \times 1,000
48 \times 100 ___ 48 \times 10 \times 10 \times 10

©White Rose Maths
### Multiply by 10, 100 and 1,000

#### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Rosie has £300 in her bank account. Tommy has 100 times more than Rosie in his bank account. How much more money does Tommy have than Rosie?</th>
<th>Tommy has £30,000&lt;br&gt;Tommy has £29,700 more than Rosie.</th>
<th>Jack is thinking of a 3-digit number. When he multiplies his number by 100, the ten thousands and hundreds digit are the same.&lt;br&gt;The sum of the digits is 10&lt;br&gt;What number could Jack be thinking of?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>181&lt;br&gt;262&lt;br&gt;343&lt;br&gt;424&lt;br&gt;505</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Whitney has £1,020 in her bank account. Tommy has £120 in his bank account. Whitney says, I have ten times more money than you.</th>
<th>Whitney is incorrect, she would need to have £1,200 if this were the case (Or Tommy would need to be £102).</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Is Whitney correct? Explain your reasoning.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Divide by 10, 100 and 1,000

Notes and Guidance

Children look at dividing by 10, 100 and 1,000 using a place value chart.

They use counters and digits to learn that the digits move to the right when dividing by powers of ten. They develop understanding of how many places to the right to move the counters to the right.

Mathematical Talk

What happens to the digits?

How are dividing by 10, 100 and 1,000 related to each other?

How are dividing by 10, 100 and 1,000 linked to multiplying by 10, 100 and 1,000?

What does ‘inverse’ mean?

Varied Fluency

What number is represented in the place value grid?

Divide the number by 100

Which direction do the counters move?

How many columns do they move? How do you know how many columns to move?

What number do we have now?

Complete the following using a place value grid.

- Divide 460 by 10
- Divide 5,300 by 100
- Divide 62,000 by 1,000

Divide these numbers by 10, 100 and 1,000

- 80,000
- 300,000
- 547,000

Calculate 45,000 ÷ 10 ÷ 10

How else could you calculate this?
Mo has £357,000 in his bank.
He divides the amount by 1,000 and takes that much money out of the bank.
Using the money he has taken out, he buys some furniture costing two hundred and sixty-nine pounds.
How much money does Mo have left from the money he took out?
Show your working out.

$$357,000 \div 1,000 = 357$$
If you subtract £269, he is left with £88

Here are the answers to some problems:

```
5,700  405  397  6,203
```

Possible solutions:

- $$3,970 \div 10 = 397$$
- $$57,000 \div 10 = 5,700$$
- $$397,000 \div 1,000 = 397$$
- $$40,500 \div 100 = 405$$
- $$620,300 \div 100 = 6,203$$

Can you write at least two questions for each answer involving dividing by 10, 100 or 1,000?
Multiples of 10, 100 and 1,000

Notes and Guidance

Children have been taught how to multiply and divide by 10, 100 and 1,000.

They now use knowledge of other multiples of 10, 100 and 1,000 to answer related questions.

Mathematical Talk

If we are multiplying by 20, can we break it down into two steps and use our knowledge of multiplying by 10?

How does using multiplication and division as the inverse of the other help us to use known facts?

Varied Fluency

36 × 5 = 180

Use this fact to solve the following questions:

36 × 50 = ____
500 × 36 = ____
5 × 360 = ____
360 × 500 = ____

Here are two methods to solve 24 × 20

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 × 10 × 2</td>
<td>24 × 2 × 10</td>
</tr>
<tr>
<td>= 240 × 2</td>
<td>= 48 × 10</td>
</tr>
<tr>
<td>= 480</td>
<td>= 480</td>
</tr>
</tbody>
</table>

What is the same about the methods, what is different?

The division diagram shows 7,200 ÷ 200 = 36

Use the diagram to solve:

3,600 ÷ 200 = □
18,000 ÷ 200 = □
5,400 ÷ □ = 27
□ = 6,600 ÷ 200
Multiples of 10, 100 and 1,000

Reasoning and Problem Solving

Tommy has answered a question. Here is his working out.

600 ÷ 25
600 ÷ 2 = 300
300 ÷ 5 = 60
60 ÷ 25 = 60

Is he correct?
Explain your answer.

Tommy is not correct as he has partitioned 25 incorrectly.

He could have divided by 5 twice.
The correct answer should be 24

6 × 7 = 42
Alex uses this multiplication fact to solve
420 ÷ 70 = ___

Alex says,

Do you agree with Alex?
Explain your answer.

Alex is wrong; both numbers (the dividend and divisor) are 10 times bigger than the numbers in the multiplication so the answer is 6.

6 × 70 = 420, therefore 420 ÷ 70 = 6

The answer is 60 because all of the numbers are 10 times bigger.
Four operations

Theme 4 – Multiplication
Multiply 4-digits by 1-digit

Notes and Guidance

Children build on previous steps to represent a 4-digit number multiplied by a 1-digit number using concrete manipulatives. Teachers should be aware of misconceptions arising from using 0 as a place holder in the hundreds, tens or ones column. Children then move on to explore multiplication with exchange in one, and then more than one column.

Mathematical Talk

Why is it important to set out multiplication using columns?

Explain the value of each digit in your calculation.

How do we show there is nothing in a place value column?

What do we do if there are ten or more counters in a place value column?

Which part of the multiplication is the product?

Varied Fluency

Complete the calculation.

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>10</td>
<td>1</td>
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<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
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</table>

Write the multiplication calculation represented and find the answer.

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<tr>
<th>Thousands</th>
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<th>Tens</th>
<th>Ones</th>
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<td>0</td>
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</table>

Remember if there are ten or more counters in a column, you need to make an exchange.

Annie earns £1,325 per week. How much would he earn in 4 weeks?

<table>
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<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
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<tr>
<td>1</td>
<td>3</td>
<td>2</td>
<td>5</td>
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</tbody>
</table>
Multiply 4-digits by 1-digit

Reasoning and Problem Solving

Alex calculated $1,432 \times 4$

Here is her answer.

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<tr>
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<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
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</tbody>
</table>

$\times$ 4

| 4 | 16 | 12 | 8 |

$1,432 \times 4 = 416,128$

Can you explain what Alex has done wrong?

Alex has not exchanged when she has got 10 or more in the tens and hundreds columns.

Can you work out the missing numbers using the clues?

$2,345 \times 5 = 11,725$

- The 4 digits being multiplied by 5 are consecutive numbers.
- The first 2 digits of the product are the same.
- The fourth and fifth digits of the answer add to make the third.
**Multiply 2-digits (Area Model)**

**Notes and Guidance**

Children use Base 10 to represent the area model of multiplication, which will enable them to see the size and scale linked to multiplying.

Children will then move on to representing multiplication more abstractly with place value counters and then numbers.

**Mathematical Talk**

What are we multiplying?
How can we partition these numbers?

Where can we see $20 \times 20$?
What does the 40 represent?

What's the same and what's different between the three representations (Base 10, place value counters, grid)?

**Varied Fluency**

Whitney uses Base 10 to calculate $23 \times 22$

How could you adapt your Base 10 model to calculate these:

$32 \times 24$

$25 \times 32$

$35 \times 32$

Rosie adapts the Base 10 method to calculate $44 \times 32$

Compare using place value counters and a grid to calculate:

$45 \times 42$

$52 \times 24$

$34 \times 43$
### Multiply 2-digits (Area Model)

#### Reasoning and Problem Solving

| Eva says, | Eva's calculation does not include $20 \times 7$ and $50 \times 3$ Children can show this with concrete or pictorial representations. | Farmer Ron has a field that measures 53 m long and 25 m wide. Farmer Annie has a field that measures 52 m long and 26 m wide. Dora thinks that they will have the same area because the numbers have only changed by one digit each. Do you agree? Prove it. | Dora is wrong. Children may prove this with concrete or pictorial representations. |

What mistake has Eva made? Explain your answer.

| Amir hasn’t finished his calculation. Complete the missing information and record the calculation with an answer. | Amir needs 8 more hundreds, $40 \times 40 = 1,600$ and he only has 800 His calculation is $42 \times 46 = 1,932$ | |

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<th>40</th>
<th>2</th>
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<tbody>
<tr>
<td>40</td>
<td>![40x]</td>
<td>![2x]</td>
</tr>
<tr>
<td>6</td>
<td>![6x]</td>
<td>![2x]</td>
</tr>
</tbody>
</table>

35
**Multiply 2-digits by 2-digits**

**Notes and Guidance**

Children will move on from the area model and work towards more formal multiplication methods.

They will start by exploring the role of the zero in the column method and understand its importance.

Children should understand what is happening within each step of the calculation process.

**Mathematical Talk**

Why is the zero important?

What numbers are being multiplied in the first line and in the second line?

When do we need to make an exchange?

What can we exchange if the product is 42 ones?

If we know what $38 \times 12$ is equal to, how else could we work out $39 \times 12$?

**Varied Fluency**

- Complete the calculation to work out $23 \times 14$
  
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<tbody>
<tr>
<td>x</td>
<td>1</td>
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<tr>
<td>9</td>
<td>2</td>
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</tr>
<tr>
<td>2</td>
<td>3</td>
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</tbody>
</table>

  Use this method to calculate:
  
  $34 \times 26$  $58 \times 15$  $72 \times 35$

- Complete to solve the calculation.

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<tr>
<td>x</td>
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<td>---</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
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</tbody>
</table>

  Use this method to calculate:
  
  $27 \times 39$  $46 \times 55$  $94 \times 49$

- Calculate:

  $38 \times 12$  $39 \times 12$  $38 \times 11$

  What's the same? What's different?
Tommy says,

It is not possible to make 999 by multiplying two 2-digit numbers.

Do you agree? Explain your answer.

Children may use a trial and error approach during which they'll further develop their multiplication skills. They will find that Tommy is wrong because $27 \times 37$ is equal to 999.

Amir has multiplied 47 by 36

<table>
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<tr>
<th></th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
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</tbody>
</table>

Alex says,

Amir is wrong because the answer should be 1,692 not 323.

Who is correct? What mistake has been made?

Alex is correct. Amir has forgotten to use zero as a place holder when multiplying by 3 tens.
Multiply 3-digits by 2-digits

Notes and Guidance
Children will extend their multiplication skills to multiplying 3-digit numbers by 2-digit numbers. They will use multiplication to find area and solve multi-step problems. Methods previously explored are still useful e.g. using an area model.

Varied Fluency

Complete:

<table>
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<tr>
<th></th>
<th>1</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td>1</td>
<td>4</td>
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<td></td>
<td>5</td>
<td>2</td>
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</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Use this method to calculate:
(132 × 4) 264 × 14 264 × 28
(132 × 10)

What do you notice about your answers?

Calculate:

637 × 24
573 × 28
573 × 82

A playground is 128 yards by 73 yards.

Calculate the area of the playground.

Mathematical Talk

Why is the zero important?

What numbers are being multiplied in the first line and the second line?

When do we need to make an exchange?

What happens if there is an exchange in the last step of the calculation?
Multiply 3-digits by 2-digits

Reasoning and Problem Solving

The pattern stops at up to \(28 \times 111\) because exchanges need to take place in the addition step.

What do you think the answer to \(25 \times 111\) will be?

What do you notice?

Does this always work?

Here are examples of Dexter’s maths work.

\[
\begin{array}{c|ccc}
\times & 9 & 8 & 7 \\
\hline
5 & 45 & 62 & 2 \\
6 & 54 & 48 & 0 \\
\hline
1 & 12 & 8 & 1 \\
\end{array}
\]

In his first calculation, Dexter has forgotten to use a zero when multiplying by 7 tens. It should have been \(987 \times 76 = 75,012\)

In the second calculation, Dexter has not included his final exchanges. 
\(324 \times 8 = 2,592\)
\(324 \times 70 = 22,680\)
The final answer should have been \(25,272\)

Pencils come in boxes of 64
A school bought 270 boxes.
Rulers come in packs of 46
A school bought 720 packs.
How many more rulers were ordered than pencils?

15,840

Can you spot it and explain why it’s wrong?

Correct each calculation.
Multiply 4-digits by 2-digits

Notes and Guidance

Children will build on their understanding of multiplying a 3-digit number by a 2-digit number and apply this to multiplying 4-digit numbers by 2-digit numbers.

It is important that children understand the steps taken when using this multiplication method.

Methods previously explored are still useful e.g. grid.

Mathematical Talk

Explain the steps followed when using this multiplication method.

Look at the numbers in each question, can they help you estimate which answer will be the largest?

Explain why there is a 9 in the thousands column.

Why do we write the larger number above the smaller number?

What links can you see between these questions? How can you use these to support your answers?

Varied Fluency

Use the method shown to calculate $2,456 \times 34$

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>2</th>
<th>5</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\times$</td>
<td>2</td>
<td>6</td>
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<tr>
<td></td>
<td>8</td>
<td>4</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

Calculate

$3,282 \times 32$ $7,132 \times 21$ $9,708 \times 38$

Use $<$, $>$ or $=$ to make the statements correct.

$4,458 \times 56$ $4,523 \times 54$

$4,458 \times 55$ $4,523 \times 54$

$4,458 \times 55$ $4,522 \times 54$
Multiply 4-digits by 2-digits

Reasoning and Problem Solving

Spot the Mistakes

Can you spot and correct the errors in the calculation?

```
  2 5 3 4
× 2 3
```

There are 2 errors. In the first line of working, the exchanged ten has not been added. In the second line of working, the place holder is missing. The correct answer should be 58,282

Teddy has spilt some paint on his calculation.

```
  2 6 9
× 2
```

What are the missing digits?

What do you notice?

The missing digits are all 8
Multiply 4-digits by 2-digits

Notes and Guidance

Children consolidate their knowledge of column multiplication, multiplying numbers with up to 4 digits by a 2-digit number. It may be useful to revise multiplication by a single digit first, and then 2- and 3-digit numbers before moving on when ready to the largest calculations. They use these skills to solve multi-step problems in a range of contexts.

Varied Fluency

Calculate.

<table>
<thead>
<tr>
<th>4</th>
<th>2</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>×</td>
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<tbody>
<tr>
<td>×</td>
<td>7</td>
<td>3</td>
<td></td>
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</tbody>
</table>

5,734 × 26

Jack made cookies for a bake sale. He made 345 cookies. The recipe says that he should have 17 raisins in each cookie.

How many raisins did he use altogether?

Work out the missing number.

6 × 35 = ____ × 5

Mathematical Talk

What is important to remember as we begin multiplying by the tens number?

How would you draw the calculation?

Can the inverse operation be used?

Is there a different strategy that you could use?
Multiply 4-digits by 2-digits

Reasoning and Problem Solving

True or False?

- $5463 \times 18 = 18 \times 5463$
  - True

- I can find the answer to $1100 \times 28$ by calculating $1100 \times 30$ and subtracting 2 lots of 1100
  - True

- $702 \times 9 = 701 \times 10$
  - False

Place the digits in the boxes to make the largest product.

\[
\begin{array}{c}
\times \\
\hline
\end{array}
\]

- $234578 \times 75$
- $632\,000$

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Four operations

Theme 5 – Factors
Factors

Notes and Guidance

Children understand the relationship between multiplication and division and use arrays to show the relationship between them. Children learn that factors of a number multiply together to give that number, meaning that factors come in pairs. Factors are the whole numbers that you multiply together to get another whole number (factor \( \times \) factor = product).

Mathematical Talk

How can you work in a systematic way to prove you have found all the factors?

Do factors always come in pairs?

How can we use our multiplication and division facts to find factors?

Varied Fluency

If you have twenty counters, how many different ways of arranging them can you find?

How many factors of twenty have you found by arranging your counters in different arrays?

Circle the factors of 60

9, 6, 8, 4, 12, 5, 60, 15, 45

Which factors of 60 are not shown?

Fill in the missing factors of 24

\[ 1 \times \_ \_ \_ \_ \_ \_ \times 12 \]

\[ 3 \times \_ \_ \_ \_ \_ \_ \times \_ \_ \_ \_ \_ \_ \]

What do you notice about the order of the factors?

Use this method to find the factors of 42
Factors

Reasoning and Problem Solving

Here is Annie’s method for finding factor pairs of 36

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>X</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

When do you put a cross next to a number?

How many factors does 36 have?

Use Annie’s method to find all the factors of 64

If it is not a factor, put a cross.

36 has 9 factors.

Factors of 64:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>X</td>
</tr>
<tr>
<td>6</td>
<td>X</td>
</tr>
<tr>
<td>7</td>
<td>X</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Always, Sometimes, Never

- An even number has an even amount of factors.
- An odd number has an odd amount of factors.

Sometimes, e.g. 6 has four factors but 36 has nine.

Sometimes, e.g. 21 has four factors but 25 has three.

True or False?

The bigger the number, the more factors it has.

False. For example, 12 has 6 factors but 13 only has 2
Common Factors

Notes and Guidance

Using their knowledge of factors, children find the common factors of two numbers.

They use arrays to compare the factors of a number and use Venn diagrams to show their results.

Mathematical Talk

How can we find the common factors systematically?

Which number is a common factor of a pair of numbers?

How does a Venn diagram help to show common factors?

Varied Fluency

Use arrays to find the common factors of 12 and 15
Can we arrange each number in counters in one row?

Yes- so they have a common factor of one.
Can we arrange each number in counters in two equal rows?

We can for 12, so 2 is a factor of 12, but we can't for 15, so 2 is not a factor of 15, meaning 2 is not a common factor of 12 and 15
Continue to work through the factors systematically until you find all the common factors.

Fill in the Venn diagram to show the factors of 20 and 24

Where are the common factors of 20 and 24?

Use a Venn diagram to show the common factors of 9 and 15
Common Factors

Reasoning and Problem Solving

**True or False?**

- 1 is a factor of every number. True
- 1 is a multiple of every number. False
- 0 is a factor of every number. False
- 0 is a multiple of every number. True

I am thinking of two 2-digit numbers. Both of the numbers have a digit total of six. Their common factors are: 1, 2, 3, 4, 6, and 12. What are the numbers? 24 and 60
Common Factors

Notes and Guidance

Children find the common factors of two numbers.

Some children may still need to use arrays and other representations at this stage but mental methods and knowledge of multiples should be encouraged.

They can show their results using Venn diagrams and tables.

Mathematical Talk

How do you know you have found all the factors of a given number?

Have you used a systematic approach?

Can you explain your system to a partner?

How does a Venn diagram show common factors?

Where are the common factors?

Varied Fluency

Find the common factors of each pair of numbers.

- 24 and 36
- 20 and 30
- 28 and 45

Which number's factors make it the odd one out?

12, 30, 54, 42, 32, 48

Can you explain why?

Two numbers have common factors of 4 and 9.

What could the numbers be?
There are 49 pears and 56 oranges.

They need to be put into baskets of pears and baskets of oranges with an equal number of fruit in each basket.

Amir says, 

There will be 8 pieces of fruit in each basket.

Jack says, 

There will be 7 pieces of fruit in each basket.

Who is correct? Explain how you know.

Jack is correct. There will be seven pieces of fruit in each basket because 7 is a common factor of 49 and 56.

Tommy has two pieces of string.

One is 160 cm long and the other is 200 cm long.

He cuts them into pieces of equal length.

What are the possible lengths the pieces of string could be?

Dora has 32 football cards that she is giving away to his friends.

She shares them equally between her friends.

How many friends could Dora have?

The possible lengths are: 2, 4, 5, 8, 10, 20 and 40 cm.

Dora could have 1, 2, 4, 8, 16 or 32 friends.
Divide 4-digits by 1-digit

Notes and Guidance

Children use their knowledge from Year 4 of dividing 3-digits numbers by a 1-digit number to divide up to 4-digit numbers by a 1-digit number.

They use place value counters to partition their number and then group to develop their understanding of the short division method.

Mathematical Talk

How many groups of 4 thousands are there in 4 thousands?
How many groups of 4 hundreds are there in 8 hundreds?
How many groups of 4 tens are there in 9 tens?
What can we do with the remaining ten?
How many groups of 4 ones are there in 12 ones?

Do I need to solve both calculations to compare the divisions?

Varied Fluency

Here is a method to calculate 4,892 divided by 4 using place value counters and short division.

Use this method to calculate:
6,610 ÷ 5  
2,472 ÷ 3  
9,360 ÷ 4

Mr Porter has saved £8,934
He shares it equally between his three grandchildren.
How much do they each receive?

Use <, > or = to make the statements correct.

3,495 ÷ 5  
3,495 ÷ 3

8,064 ÷ 7  
9,198 ÷ 7

7,428 ÷ 4  
5,685 ÷ 5
Divide 4-digits by 1-digit

Reasoning and Problem Solving

Jack is calculating $2,240 \div 7$

He says you can’t do it because 7 is larger than all of the digits in the number.

Do you agree with Jack? Explain your answer.

Jack is incorrect. You can exchange between columns. You can’t make a group of 7 thousands out of 2 thousand, but you can make groups of 7 hundreds out of 22 hundreds.

The answer is 320
**Divide with Remainders**

**Notes and Guidance**

Children continue to use place value counters to partition and then group their number to further develop their understanding of the short division method.

They start to focus on remainders and build on their learning from Year 4 to understand remainders in context. They do not represent their remainder as a fraction at this point.

**Mathematical Talk**

If we can’t make a group in this column, what do we do?
What happens if we can’t group the ones equally?
In this number story, what does the remainder mean?
When would we round the remainder up or down?
In which context would we just focus on the remainder?

**Varied Fluency**

Here is a method to solve 4,894 divided by 4 using place value counters and short division.

<table>
<thead>
<tr>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8</td>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>

Use this method to calculate:

- 6,613 ÷ 5
- 2,471 ÷ 3
- 9,363 ÷ 4

- Muffins are packed in trays of 6 in a factory. In one day, the factory makes 5,623 muffins. How many trays do they need? How many trays will be full? Why are your answers different?

- For the calculation 8,035 ÷ 4
  - Write a number story where you round the remainder up.
  - Write a number story where you round the remainder down.
  - Write a number story where you have to find the remainder.
### Divide with Remainders

#### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>I am thinking of a 3-digit number.</th>
<th>Possible answers:</th>
<th>Sometimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>When it is divided by 9, the remainder is 3</td>
<td>129, 219</td>
<td>Possible answers:</td>
</tr>
<tr>
<td>When it is divided by 2, the remainder is 1</td>
<td>309, 399</td>
<td>432 ÷ 1 = 432 r 0</td>
</tr>
<tr>
<td>When it is divided by 5, the remainder is 4</td>
<td>489, 579</td>
<td>543 ÷ 2 = 271 r 1</td>
</tr>
<tr>
<td></td>
<td>669, 759</td>
<td>654 ÷ 3 = 218 r 0</td>
</tr>
<tr>
<td></td>
<td>849, 939</td>
<td>765 ÷ 4 = 191 r 1</td>
</tr>
</tbody>
</table>

Encourage children to think about the properties of numbers that work for each individual statement. This will help decide the best starting point.

#### Always, Sometimes, Never

A three-digit number made of consecutive descending digits divided by the next descending digit always has a remainder of 1

**765 ÷ 4 = 191 remainder 1**

How many possible examples can you find?

- 432 ÷ 1 = 432 r 0
- 543 ÷ 2 = 271 r 1
- 654 ÷ 3 = 218 r 0
- 765 ÷ 4 = 191 r 1
- 876 ÷ 5 = 175 r 1
- 987 ÷ 6 = 164 r 3
Short Division

Notes and Guidance

Children build on their understanding of dividing up to 4-digits by 1-digit by now dividing by up to 2-digits. They use the short division method and focus on the grouping structure of division. Teachers may encourage children to list multiples of the divisor (number that we are dividing by) to help them solve the division more easily. Children should experience contexts where the answer “4 r 1” means both 4 complete boxes or 5 boxes will be needed.

Mathematical Talk

In the hundreds column, how many groups of 5 are in 7? Are there any hundreds remaining? What do we do next?

In the thousands column, there are no groups of three in 1. What do we do?

Why is the context of the question important when deciding how to round the remainders after a division?

Varied Fluency

Calculate using short division.

\[
\begin{array}{c}
5 \quad 7 \quad 2 \quad 5 \\
\hline
3 \quad 1 \quad 9 \quad 3 \quad 8
\end{array}
\]

\[
\begin{array}{c}
1 \quad 2 \quad 6 \quad 0 \quad 3 \quad 6 \\
\hline
3,612 \div 14
\end{array}
\]

List the multiples of the divisors to help you calculate.

A limousine company allows 14 people per limousine.

How many limousines are needed for 230 people?

Year 6 has 2,356 pencil crayons for the year.

They put them in bundles, with 12 in each bundle.

How many complete bundles can be made?
Short Division

Reasoning and Problem Solving

Find the missing digits.

\[
\begin{array}{c}
\frac{0 \div 4 \div 1 \div 4 \div r \div 3}{41 \div 6 \div 5 \div 9}
\end{array}
\]

Work out the value of C. (The bar models are not drawn to scale)

4,950 ÷ 3 = 1,650
1,650 ÷ 3 = 550
550 ÷ 5 = 110

Here are two calculations.

\[
\begin{array}{c}
A = 396 \div 11 \\
B = 832 \div 13
\end{array}
\]

396 ÷ 11 = 36
832 ÷ 13 = 64
64 − 36 = 28

Find the difference between A and B.
Division using Factors

Notes and Guidance

Children use their number sense, specifically their knowledge of factors, to be able to see relationships between the dividend (number being divided) and the divisor (number that the dividend is being divided by).

Beginning with multiples of 10 will allow children to see these relationships, before moving to other multiples.

Mathematical Talk

What is a factor?
How does using factor pairs help us to answer division questions?
Do you notice any patterns?
Does using factor pairs always work?
Is there more than one way to solve a calculation using factor pairs?
What methods can be used to check your working out?

Varied Fluency

Calculate 780 ÷ 20
Now calculate 780 ÷ 10 ÷ 2
What do you notice? Why does this work?
Use the same method to calculate 480 ÷ 60

Use factors to help you calculate.

4,320 ÷ 15

Eggs are put into boxes. Each box holds 12 eggs. A farmer has 648 eggs that need to go in the boxes.

How many boxes will he fill?
### Division using Factors

#### Reasoning and Problem Solving

| Calculate:          | 26  
|---------------------|------
|                     | 52   
|                     | 104  
| What did you do each time? What was your strategy? What do you notice? Why? | Class 6 are calculating $7,848 \div 24$  
| Tommy says,         |  
| Tommy is wrong: he has partitioned 15 when he should have used factor pairs. He could have used factor pairs 5 and 3 and divided by 5 then 3 (or 3 then 5). | The children decide which factor pairs to use. Here are some of their suggestions:  
| Do you agree? Explain why. |  
|                     | 2 and 12  
|                     | 1 and 24   
|                     | 4 and 6  
|                     | 10 and 14   
|                     | Which will not give them the correct answer? Why? | 10 and 14 is incorrect because they are not factors of 24 (to get 10 and 14, 24 has been partitioned).  
|                     | The correct answer is 327 |  
|                     | Children should get the same answer using all 3 factor pairs methods. |  
|                     | Using the factor pair of 1 and 24 is the least efficient. |  

| 1,248 ÷ 48  
| 1,248 ÷ 24  
| 1,248 ÷ 12 |
Long Division (1)
Notes and Guidance

Children are introduced to long division as a different method of dividing by a 2-digit number.

They divide 3-digit numbers by a 2-digit number without remainders, starting with a more expanded method (with multiples shown), before progressing to the more formal long division method.

Mathematical Talk

How can we use multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided.)

Varied Fluency

Use this method to calculate:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>−</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>−</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Multiples of 12:

|   | 12 × 1 = 12 |
|   | 12 × 2 = 24 |
|   | 12 × 3 = 36 |
|   | 12 × 4 = 48 |
|   | 12 × 5 = 60 |
|   | 12 × 6 = 72 |
|   | 12 × 7 = 84 |
|   | 12 × 8 = 96 |
|   | 12 × 7 = 108 |
|   | 12 × 10 = 120 |

Use the long division method to calculate:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>−</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>−</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

836 ÷ 11
798 ÷ 14
608 ÷ 19
Long Division (1)

Reasoning and Problem Solving

Odd One Out

Which is the odd one out?
Explain your answer.

512 ÷ 16
672 ÷ 21
792 ÷ 24

792 ÷ 24 = 33 so this is the odd one out as the other two give an answer of 32

Spot the Mistake

855 ÷ 15 =

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>5</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>8</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>-</td>
<td>7</td>
<td>5</td>
<td>(× 4)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>(× 10)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

The mistake is that 105 ÷ 15 is not equal to 10

105 ÷ 15 = 7 so the answer to the calculation is 57
**Long Division (2)**

**Notes and Guidance**

Building on using long division with 3-digit numbers, children divide 4-digit numbers by 2-digits using the long division method.

They use their knowledge of multiples and multiplying and dividing by 10 and 100 to calculate more efficiently.

**Mathematical Talk**

How can we use multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided).

**Varied Fluency**

Here is a division method.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>7</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use this method to calculate:

2,208 ÷ 16  
1,755 ÷ 45  
1,536 ÷ 16

There are 1,989 footballers in a tournament. Each team has 11 players and 2 substitutes. How many teams are there in the tournament?
Long Division (2)

Reasoning and Problem Solving

Which calculation is harder?

1,950 ÷ 13

1,950 ÷ 15

Explain why.

Dividing by 13 is harder because 13 is prime so we cannot use factor knowledge to factorise it into smaller parts. The 13 times table is harder than the 15 times table because the 15 times table is related to the 5 times table whereas the 13 times table is not related to a more common times table (because 13 is prime).

6,120 ÷ 17 = 360

Explain how to use this fact to find

6,480 ÷ = 360

6,480 is 360 more than 6,120, so there is 1 group of 360 more.

Therefore, there are 18 groups of 360, so the answer is 18
Long Division (3)

Notes and Guidance

Children now divide using long division where answers have remainders. After dividing, they check that the remainder is smaller than the divisor.

Children start to understand how to interpret the remainder e.g. $380 \div 12 = 31 \text{ r } 8$ could mean 31 full packs, or 32 packs needed depending on context.

Mathematical Talk

How can we use multiples to help us divide?

What happens if we cannot divide the ones exactly by the divisor? How do we show what is left over?

Why are we subtracting the totals from the dividend (starting number)?

Why is the context of the question important when deciding how to round the remainders after a division?

Varied Fluency

Tommy uses this method to calculate 372 divided by 15. He has used his knowledge of multiples to help.

\[
\begin{array}{cccc|c}
1 & 5 & 3 & 7 & 2 \\
\hline
2 & 4 & r & 1 & 2 \\
1 & 15 & = & 15 \\
2 & 15 & = & 30 \\
3 & 15 & = & 45 \\
4 & 15 & = & 60 \\
5 & 15 & = & 75 \\
10 & 15 & = & 150 \\
\end{array}
\]

Use this method to calculate:

\[
\begin{align*}
271 \div 17 & \\
623 \div 21 & \\
842 \div 32 & \\
\end{align*}
\]

A school needs to buy 380 biscuits for parents' evening. Biscuits are sold in packs of 12.

How many packets will the school need to buy?
Long Division (3)

Reasoning and Problem Solving

Here are two calculation cards.

- A = 396 ÷ 11
  - Rosie is correct because 832 is not a multiple of 11
    - 396 ÷ 11 = 36
    - 832 ÷ 11 = 75 r 7

Whitney thinks there won’t be a remainder for either calculation because 396 and 832 are both multiples of 11.

Rosie disagrees, she has done the written calculations and says one of them has a remainder.

Who is correct? Explain your answer.

576 children and 32 adults need transport for a school trip. A coach holds 55 people.

- Alex is correct.
  - There are 608 people altogether, 608 ÷ 55 = 11 r 3, so 12 coaches are needed.

  We need 10 coaches.

  We need 11 coaches.

  We need 12 coaches.

Who is correct? Explain how you know.

How many spare seats will there be?

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Long Division (4)

Notes and Guidance

Children now divide four-digit numbers using long division where their answers have remainders. After dividing, they check that their remainder is smaller than their divisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that it is not applicable.

Mathematical Talk

How can we use multiples to help us divide?

What happens if we cannot divide the ones exactly by the divisor? How do we show what is left over?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

Does the remainder need to be rounded up or down?

Varied Fluency

Amir used this method to calculate 1,426 divided by 13

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>0</th>
<th>9</th>
<th>r</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>−</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>−</td>
<td>1</td>
<td>1</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(x × 100)

(x × 9)

Use this method to calculate:

\[2,637 \div 16\]  \[4,453 \div 22\]  \[4,203 \div 18\]

A large bakery produces 7,849 biscuits in a day which are packed in boxes. Each box holds 64 biscuits.

How many boxes are needed so all the biscuits are in a box?
### Long Division (4)

#### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Class 6 are calculating three thousand, six hundred and thirty-three divided by twelve.</th>
<th>Rosie is correct because 3,633 is odd and 12 is even, and all multiples of 12 are even because 12 is even.</th>
<th>Which numbers up to 20 can 4,236 be divided by without having a remainder?</th>
<th>1, 2, 3, 4, 6, 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosie says that she knows there will be a remainder without calculating.</td>
<td>3,633 ÷ 12 = 302 r 9, so the remainder is 9</td>
<td>What do you notice about all the numbers?</td>
<td>They are all factors of 12</td>
</tr>
</tbody>
</table>
### Prime Numbers

**Notes and Guidance**

Using their knowledge of factors, children see that some numbers only have two factors. They are taught that these are numbers called prime numbers, and that non-primes are called composite numbers. Children can recall primes up to 19 and are able to establish whether a number is prime up to 100. Using primes, they break a number down into its prime factors. Children learn that 1 is not a prime number because it does not have exactly two factors (it only has 1 factor).

### Mathematical Talk

- How many factors does each number have?
- How many other numbers can you find that have this number of factors?
- What is a prime number?
- What is a composite number?
- How many factors does a prime number have?

### Varied Fluency

- **Use counters to find the factors of the following numbers.**
  
  5, 13, 17, 23

  What do you notice about the arrays?

- **A prime number has exactly 2 factors, one and itself. A composite number can be divided by numbers other than 1 and itself to give a whole number answer.**

  Sort the numbers into the table.

  2 3 5 9 15 24 29 30

<table>
<thead>
<tr>
<th>Prime</th>
<th>Composite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exactly 2 factors (1 and itself)</td>
<td></td>
</tr>
<tr>
<td>More than 2 factors</td>
<td></td>
</tr>
</tbody>
</table>

  Put two of your own numbers into the table. Why are two of the boxes empty? Would 1 be able to go in the tablet? Why or why not?
Prime Numbers

Reasoning and Problem Solving

Find all the prime numbers between 10 and 100, sort them in the table below.

<table>
<thead>
<tr>
<th>End in a 1</th>
<th>End in a 3</th>
<th>End in a 7</th>
<th>End in a 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>11, 31, 41, 61, 71</td>
<td>13, 23, 43, 53, 73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17, 37, 47, 67, 97</td>
<td>19, 29, 59, 79, 89</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Why do no two-digit prime numbers end in an even digit?

Because all two-digit even numbers have more than 2 factors.

Why do no two-digit prime numbers end in a 5?

Because all two-digit numbers ending in 5 are divisible by 5 as well as 1 and itself, so have more than 2 factors.

Dora says all prime numbers have to be odd.

Her friend Amir says that means all odd numbers are prime, so 9, 27 and 45 are prime numbers.

Dora is incorrect because 2 is a prime number (it has exactly 2 factors).

Amir thinks all odd numbers are prime but he is incorrect because most odd numbers have more than 2 factors.

E.g.
Factors of 9: 1, 3 and 9
Factors of 27: 1, 3, 9 and 27

Explain Amir's and Dora's mistakes and correct them.
Square Numbers

Notes and Guidance
Children will need to be able to find factors of numbers. Square numbers have an odd number of factors and are the result of multiplying a whole number by itself.

Children learn the notation for squared is

Mathematical Talk

Why are square numbers called ‘square’ numbers?
Are there any patterns in the sequence of square numbers?
Are the squares of even numbers always even?
Are the squares of odd numbers always odd?

Varied Fluency

What does this array show you?
Why is this array square?

How many ways are there of arranging 36 counters in an array?
What is the same about each array?
What is different?

Find the first 12 square numbers.
Show why they are square numbers.
How many different squares can you make using counters?
What do you notice?
Are there any patterns?
**Square Numbers**

**Reasoning and Problem Solving**

<table>
<thead>
<tr>
<th>Teddy says,</th>
<th>No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors come in pairs so all numbers must have an even number of factors.</td>
<td>Square numbers have an odd number of factors (e.g. the factors of 25 are 1, 25 and 5).</td>
</tr>
</tbody>
</table>

Do you agree? Explain your reasoning.

<table>
<thead>
<tr>
<th>How many square numbers can you make by adding prime numbers together?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Here’s one to get you started:</td>
</tr>
<tr>
<td>2 + 2 = 4</td>
</tr>
</tbody>
</table>

Solutions include:
- 2 + 2 = 4
- 2 + 7 = 9
- 11 + 5 = 16
- 23 + 2 = 25
- 29 + 7 = 36

**Whitney thinks that 4² is equal to 16**

Do you agree? Convince me.

**Amir thinks that 6² is equal to 12**

Do you agree? Explain what you have noticed.

**Always, Sometimes, Never**

A square number has an even number of factors.

Children may use concrete materials or draw pictures to prove it. Children should spot that 6 has been multiplied by 2. They may create the array to prove that 6² = 36 and 6 × 2 = 12.

Never. Square numbers have an odd number of factors because one of their factors does not have a pair.
Cube Numbers

Notes and Guidance

Children learn that a cubenumber is the result of multiplying a whole number by itself three times e.g. $6 \times 6 \times 6$

If you multiply a number by itself, then itself again, the result is a cubenumber.

Children learn the notation for cubed is $3^3$

Mathematical Talk

Why are cube numbers called ‘cube’ numbers?

How are squared and cubed numbers similar?

How are they different?

True or False: cubes of even numbers are even and cubes of odd numbers are odd.

Varied Fluency

Use multilink cubes to investigate how many are needed to make different sized cubes.

How many multilink blocks are required to make the first cube number? The second? Third?

Can you predict what the tenth cube number is going to be?

Complete the table.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^3$</td>
<td>$3 \times 3 \times 3$</td>
<td>27</td>
</tr>
<tr>
<td>$4^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5^3$</td>
<td>$5 \times 5 \times 5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$6 \times 6 \times 6$</td>
<td></td>
</tr>
</tbody>
</table>

Calculate:

$4^3 = ____$

$5^3 = ____$

$3$ cubed $= ____$

$6$ cubed $= ____$
## Cube Numbers

### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Rosie says,</th>
<th>Rosie is wrong, she has multiplied 5 by 3 rather than by itself 3 times. $5^3 = 5 \times 5 \times 5$ $5 \times 5 \times 5 = 125$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5³ is equal to 15</td>
<td>Do you agree? Explain your answer.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Here are 3 cards</th>
<th>A = 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>64</td>
</tr>
<tr>
<td>C</td>
<td>125</td>
</tr>
</tbody>
</table>

On each card there is a cube number. Use these calculations to find each number.

- $A \times A = B$
- $B + B - 3 = C$
- Digit total of $C = A$

<table>
<thead>
<tr>
<th>Dora is thinking of a two-digit number that is both a square and a cube number. What number is she thinking of?</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teddy’s age is a cube number. Next year his age will be a square number. How old is he now?</td>
<td>8 years old</td>
</tr>
<tr>
<td>The sum of a cube number and a square number is 150. What are the two numbers?</td>
<td>125 and 25</td>
</tr>
</tbody>
</table>
Primes to 100

Notes and Guidance

Building on their learning in year 5, children should know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

They should be able to use their understanding of prime numbers to work out whether or not numbers up to 100 are prime. Using primes, they break a number down into its prime factors.

Mathematical Talk

What is a prime number?
What is a composite number?
How many factors does a prime number have?
Are all prime numbers odd?
Why is 1 not a prime number?
Why is 2 a prime number?

Varied Fluency

- List all of the prime numbers between 10 and 30
- The sum of two prime numbers is 36
  What are the numbers?
- All numbers can be broken down into prime factors.
  A prime factor tree can help us find them.
  Complete the prime factor tree for 20

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### Primes to 100

#### Reasoning and Problem Solving

**Use the clues to work out the number.**
- It is greater than 10
- It is an odd number
- It is not a prime number
- It is less than 25
- It is a factor of 60

**Shade in the multiples of 6 on a 100 square.**

What do you notice about the numbers either side of every multiple of 6?

**Eva says,**

I noticed there is always a prime number next to a multiple of 6.

**Is she correct?**

**Both numbers are always odd.**

Yes, Eva is correct because at least one of the numbers either side of a multiple of 6 is always prime for numbers up to 100.
Square & Cube Numbers

Notes and Guidance

Children have identified square and cube numbers previously and now explore the relationship between them, and solve problems involving them. They need to experience sorting the numbers into different diagrams and look for patterns and relationships. They explore general statements regarding square and cube numbers. This step is a good opportunity to practise efficient mental methods of calculation.

Mathematical Talk

What do you notice about the sequence of square numbers?

What do you notice about the sequence of cube numbers?

Explore the pattern of the difference between the numbers.

### Varied Fluency

- Use <, > or = to make the statements correct.
  - 3 cubed  
  - 4 squared
  - 8 squared 
  - 4 cubed
  - 11 squared
  - 5 cubed

- This table shows square and cube numbers. Complete the table. Explain the relationships you can see between the numbers.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 x 3</td>
<td>3^3</td>
<td>8</td>
</tr>
<tr>
<td>4 x 4</td>
<td>4 x 4 x 4</td>
<td>27</td>
</tr>
<tr>
<td>25</td>
<td>5^3</td>
<td>6 x 6 x 6</td>
</tr>
<tr>
<td>8^2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- ____ + 35 = 99
- 210 - ____ = 41
- Which square numbers are missing from the calculations?
## Square & Cube Numbers

### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Place 5 odd and 5 even numbers in the table.</th>
<th>Possible cube numbers to use: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000</th>
<th>Shade in all the square numbers on a 100 square.</th>
<th>Square numbers are always either a multiple of 4 or 1 more than a multiple of 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Table" /></td>
<td><img src="image" alt="Table" /></td>
<td><img src="image" alt="Table" /></td>
<td><img src="image" alt="Table" /></td>
</tr>
</tbody>
</table>

**Jack says,**

The smallest number that is both a square number and a cube number is 64.

Do you agree with Jack? Explain why you agree or disagree.
Four operations

Theme 8 – Order of operations
Order of Operations

Notes and Guidance

Children will look at different operations within a calculation and consider how the order of operations affects the answer. Children will learn that, in mixed operation calculations, calculations are not carried out from left to right. Children learn the convention that when there is no operation sign written this means multiply e.g. $4(2 + 1)$ means $4 \times (2 + 1)$. This image is useful when teaching the order of operations.

Mathematical Talk

Does it make a difference if you change the order in a mixed operation calculation?

What would happen if we did not use the brackets?

Would the answer be correct?

Why?

Varied Fluency

- Alex has 7 bags with 5 sweets in each bag. She adds one more sweet to each bag. Which calculation will work out how many sweets she now has in total? Explain your answer.
  
  $7 \times (5 + 1) = 32$
  
  $7 \times 5 + 1 = 36$

- Teddy has completed this calculation and got an answer of 5.
  
  $14 - 4 \times 2 \div 4 = 5$

  Explain and correct his error.

- Add brackets and missing numbers to make the calculations correct.
  
  $6 + \_\_ \times 5 = 30$
  
  $25 - 6 \times \_\_ = 38$
Order of Operations

Reasoning and Problem Solving

**Countdown**

Big numbers: 25, 50, 75, 100

Small numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Children randomly select 6 numbers.

Reveal a target number.

Children aim to make the target number ensuring they can write it as a single calculation using order of operations.

Write different number sentences using the digits 3, 4, 5 and 8 before the equals sign that use:

- One operation
- Two operations with no brackets
- Two operations with brackets

**Possible solutions:**

58 − 34 = 24

58 + 3 × 4 = 60

5(8 − 3) + 4 = 29
Four operations

Theme 9 – Estimating
Estimate and Approximate

Notes and Guidance

Children build on their understanding of estimating and rounding to estimate answers for calculations and problems. The term approximate is used throughout.

Encourage children to consider the most appropriate number to round to e.g. the nearest ten, hundred or thousand. Reinforce the idea that an estimate should be performed quickly by choosing much easier numbers.

Mathematical Talk

Which numbers shall I round to?

Why should I round to this number?

Why should an estimate be quick?

When, in real life, would we use an estimate?

Varied Fluency

Which is best to estimate the total of 22,223 and 5,687?

- 22,300 + 5,700
- 22,200 + 5,700
- 22,200 + 5,600

Here are the attendances from the last 3 months at a rugby club.

<table>
<thead>
<tr>
<th>Month</th>
<th>Attendance</th>
</tr>
</thead>
<tbody>
<tr>
<td>February</td>
<td>18,655</td>
</tr>
<tr>
<td>March</td>
<td>31,402</td>
</tr>
<tr>
<td>April</td>
<td>27,092</td>
</tr>
</tbody>
</table>

What is the approximate total of February and March?

What is the approximate difference between March and April?

What is the approximate total of the three months?

April and May had an approximate total of 50,000

Estimate the attendance in May.
True or False?

49,999 − 19,999 = 50,000 − 20,000

True

Dora has used her related number facts. Both numbers on the right have increased by 1 therefore whatever the difference is, it will remain the same as the left hand side.

Can you explain why Dora’s method work?

Can you think of another example where this method could be used?

Which estimate is inaccurate?

B is inaccurate. The arrow is about a quarter of the way along the number line so it should be 30,000.

a) 0 — 25,000 — 100,000

b) 10,000 — 25,000 — 90,000

c) 0 — 25,000 — 50,000

Explain how you know.
**Mental Calculations**

**Notes and Guidance**

We have included this small step separately to ensure that teachers emphasise this important skill. Discussions with children around efficient mental calculations and sensible estimations need to run through all steps.

Sometimes children are too quick to move to computational methods, when more efficient mental strategies should be used.

**Mathematical Talk**

Is there an easy and quick way to do this?

Can you use known facts to answer the problem?

Can you use rounding?

Does the solution need an exact answer?

How does knowing the approximate answer help with the calculation?

**Varied Fluency**

How could you change the order of these calculations to be able to perform them mentally?

- $50 \times 16 \times 2$
- $30 \times 12 \times 2$
- $4 \times 17 \times 25$

Mo wants to buy a t-shirt for £9.99, socks for £1.49 and a belt for £8.99.

He has £22 in his wallet.

How could he quickly check if he has enough money?

What number do you estimate is shown by arrow B when:

- $A = 0$ and $C = 1,000$
- $A = 30$ and $C = 150$
- $A = -7$ and $C = 17$
- $A = 1$ and $C = 2$
- $A = 1,000$ and $C = 100,000$
Mental Calculations

Class 6 are calculating the total of 3,912 and 3,888

Alex says, We can just double 3,900

Is Alex correct? Explain.

Alex is correct because 3,912 is 12 more than 3,900 and 3,888 is 12 less than 3,900

3,900 \times 2 = 7,800

2,000 − 1,287

Here are three different strategies for this subtraction calculation:

I used the column method.

I used my number bonds from 87 to 100 then from 1,300 to 2,000

I subtracted one from each number and then used the column method.

Children share their ideas. Discuss how Dora’s method is inefficient for this calculation because of the need to make multiple exchanges.

Jack’s method is known as the ‘constant difference’ method and avoids exchanging.

Whose method is most efficient?
Four operations

Theme 10 – Related facts
Reason from Known Facts

Notes and Guidance

Children should use known facts from one calculation to determine the answer of another similar calculation without starting afresh.

They should use reasoning and apply their understanding of commutativity and inverse operations.

Mathematical Talk

What is the inverse?

When do you use the inverse?

How can we use multiplication/division facts to help us answer similar questions?

Varied Fluency

Complete.

\[ 70 \div \_\_ = 7 \quad 3.5 \times 10 = \_\_ \]

\[ 70 \div \_\_ = 3.5 \quad \_\_ = 3.5 \times 20 \]

\[ 70 \div \_\_ = 14 \quad \_\_ = 3.5 \times 2 \]

Make a similar set of calculations using \( 90 \div 2 = 45 \)

\[ 5,138 \div 14 = 367 \]

Use this to calculate \( 15 \times 367 \)

\[ 14 \times 8 = 112 \]

Use this to calculate:

- \( 1.4 \times 8 \)
- \( 9 \times 14 \)
Reason from Known Facts

Reasoning and Problem Solving

3,565 + 2,250 = 5,815

Use this calculation to decide if the following calculations are true or false.

**True or False?**

<table>
<thead>
<tr>
<th>Calculation</th>
<th>True/False</th>
</tr>
</thead>
<tbody>
<tr>
<td>4,565 + 1,250 = 5,815</td>
<td>True</td>
</tr>
<tr>
<td>5,815 − 2,250 = 3,565</td>
<td>True</td>
</tr>
<tr>
<td>4,815 − 2,565 = 2,250</td>
<td>True</td>
</tr>
<tr>
<td>3,595 + 2,220 = 5,845</td>
<td>False</td>
</tr>
</tbody>
</table>

Which calculations will give an answer that is the same as the product of 12 and 8?

- $3 \times 4 \times 8$
- $12 \times 4 \times 2$
- $2 \times 10 \times 8$

The product of 12 and 8 is 96.

The 1st and 2nd calculations give an answer of 96. In the 1st calculation, 12 has been factorised into 3 and 4, and in the 2nd calculation, 8 has been factorised into 4 and 2.

The third calculation gives an answer of 160.