Overview

Small Steps

- Multiples
- Factors
- Common factors
- Prime numbers
- Square numbers
- Cube numbers
- Multiply by 10, 100 and 1,000
- Divide by 10, 100 and 1,000
- Multiples of 10, 100 and 1,000

NC Objectives

Multiply and divide numbers mentally drawing upon known facts.

Multiply and divide whole numbers by 10, 100 and 1000.

Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.

Recognise and use square numbers and cube numbers and the notation for squared (2) and cubed (3).

Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes.

Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

Establish whether a number up to 100 is prime and recall prime numbers up to 19.
Multiples

Notes and Guidance
Building on their times tables knowledge, children will find multiples of whole numbers. Children build multiples of a number using concrete and pictorial representations e.g. in an array.

Mathematical Talk
What do you notice about the multiples of 2? What is the same about them, what is different?
Look at multiples of other numbers; is there a rule that links them?

Varied Fluency

Circle the multiples of 5

25  32  54  40  175  3000

What do you notice about the multiples of 5?

Write down all the multiples of 4 between 20 and 80

Roll 2 dice (1-6), multiply the numbers.
What is the number a multiple of?
Is it a multiple of more than one number?

How many different numbers can you make multiples of?
Can you make multiples of all numbers up to 10?
Can you make multiples of all numbers up to 20?

Use a table to show your results.
Multiply the numbers you roll to complete the table.
Multiples

Reasoning and Problem Solving

Use the digits 0 – 9. Choose 2 digits. Multiply them together.

What is your number a multiple of?

Is it a multiple of more than one number?

Can you find all the numbers you could make?

Use the table below to help.

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>1</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Always, sometimes, never.

- The product of two even numbers is a multiple of an odd number.
- The product of two odd numbers is a multiple of an even number.

Clare’s age is a multiple of 7 and is 3 less than a multiple of 8
She is younger than 40
How old is Clare?

Always - all multiples of 1

Never - Two odd numbers multiplied together are always a multiple of an odd number.

Claire is 21 years old.
Factors

Notes and Guidance

Children understand the relationship between multiplication and division and can use arrays to show the relationship between them. They know that division can involve either sharing or finding equal groups of amounts. Children learn that a factor of a number is the number you get when you divide a whole number by another whole number and that factors come in pairs. (factor \times \text{factor} = \text{product}).

Mathematical Talk

How can you work in a systematic way to prove you have found all the factors?

Do factors always come in pairs?

How can we use our multiplication and division facts to find factors?

Varied Fluency

If you have twenty counters, how many different ways of arranging them can you find?

Circle the factors of 60

9, 6, 8, 4, 12, 5, 60, 15, 45

Which factors of 60 are not shown?

Fill in the missing factors of 24

1 \times \text{___} \quad \text{___} \times 12

3 \times \text{___} \quad \text{___} \times \text{___}

What do you notice about the order of the factors?

Use this method to find the factors of 42
Factors

Reasoning and Problem Solving

Here is Kayla's method for finding factor pairs:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>X</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

When do you put a cross next to a number?

How many factors does 36 have?

Use Kayla's method to find all the factors of 64

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>X</td>
</tr>
<tr>
<td>6</td>
<td>X</td>
</tr>
<tr>
<td>7</td>
<td>X</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

If it is not a factor, put a cross.

36 has 9 factors.

Factors of 64:

Always, sometimes, never.

- An even number has an even amount of factors.
- An odd number has an odd amount of factors.

Sometimes, e.g. 6 has four factors but 36 has nine.

Sometimes, e.g. 21 has four factors but 25 has three.

True or False?

The bigger the number, the more factors it has.

False. For example, 12 has 6 factors but 97 only has 2.
Common Factors

Notes and Guidance

Using their knowledge of factors, children find the common factors of two numbers.

They use arrays to compare the factors of a number and use a Venn diagram to show their results.

Varied Fluency

Use arrays to find the common factors of 12 and 15

Can we arrange the counters in one row?

Yes - so they have a common factor of one.

Can we arrange the counters in two equal rows?

2 is a factor of 12 but not of 15 so 2 is not a common factor.

Continue to work through the factors systematically until you find all the common factors.

Fill in the Venn diagram to show the factors of 20 and 24

Where are the common factors of 20 and 24?

Can you use a Venn diagram to show the common factors of 9 and 15?

Mathematical Talk

How can we find the common factors systematically?

Which number is a common factor of any pair of numbers?

How does a Venn diagram help to show common factors?

Where are the common factors?
## Common Factors

### Reasoning and Problem Solving

#### True or False?

<table>
<thead>
<tr>
<th>Statement</th>
<th>True or False</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 is a factor of every number.</td>
<td>True</td>
</tr>
<tr>
<td>1 is a multiple of every number.</td>
<td>False</td>
</tr>
<tr>
<td>0 is a factor of every number.</td>
<td>False</td>
</tr>
<tr>
<td>0 is a multiple of every number.</td>
<td>True</td>
</tr>
</tbody>
</table>

I am thinking of two 2-digit numbers.

Both of the numbers have a digit total of six.

Their common factors are: 1, 2, 3, 4, 6, & 12

What are the numbers? 24 and 60
Prime Numbers

Notes and Guidance

Using their knowledge of factors, children see that some numbers only have 2 factors and these are special numbers called Prime Numbers. They also learn that non-primes are called composite numbers. Children can recall primes up to 19 and are able to establish whether a number is prime up to 100. Using primes, they break a number down into its prime factors.

Mathematical Talk

How many factors does each number have?

How many other numbers can you find that have this number of factors?

What is a prime number?

What is a composite number?

How many factors does a prime number have?

Varied Fluency

Use counters to find the factors of the following numbers.

5, 13, 17, 23

What do you notice about the arrays?

A prime number has 2 factors, one and itself. A composite number can be divided by numbers other than 1 and itself. Sort the numbers into the table.

<table>
<thead>
<tr>
<th>5</th>
<th>15</th>
<th>9</th>
<th>12</th>
<th>3</th>
<th>27</th>
<th>24</th>
<th>30</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Prime</th>
<th>Composite</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 factors (1 &amp; itself)</td>
<td></td>
</tr>
<tr>
<td>More than 2 factors</td>
<td></td>
</tr>
</tbody>
</table>

Put two of your own numbers into the table.

Why are two of the boxes empty?

Where would 1 go in the table? Would it fit in at all?
Prime Numbers

Reasoning and Problem Solving

Find all the prime numbers between 10 and 100, sort them in the table below.

<table>
<thead>
<tr>
<th>End in a 1</th>
<th>End in a 3</th>
<th>End in a 7</th>
<th>End in a 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>11, 31, 41, 61, 71,</td>
<td>13, 23, 43, 53, 73,</td>
<td>17, 37, 47, 67, 97,</td>
<td>19, 29, 59, 79, 89,</td>
</tr>
</tbody>
</table>

Dora says all prime numbers have to be odd.
Her friend Abdul says that means 9, 27 and 45 are prime numbers.

Explain Abdul and Dora’s mistakes and correct them.

Why do no two-digit prime numbers end in an even number?

Why do no two-digit prime numbers end in a 5?

No 2-digit primes end in an even number because 2-digit even numbers are divisible by 2.
No 2-digit prime numbers end in a 5 because they are divisible by 5 as well as 1 and itself.

2 is a prime number so Dora is wrong.
Abdul thinks all odd numbers are prime but he is wrong as some have more than 2 factors. E.g.

Factors of 9: 1, 3 & 9
Factors of 27: 1, 3, 9 & 27
Etc.
Square Numbers

Notes and Guidance

Children will need to be able to find factors of whole numbers. Square numbers have an odd number of factors and are the result of multiplying a number by itself.

They learn the notation for squared is \( \times^2 \).

Varied Fluency

- What does this array show you?
  Why is it square?

- How many ways are there of arranging 36 counters in an array?
  What is the same about each array?
  What is different?

- Find the first 12 square numbers.
  Prove that they are square numbers.
  How many different squares can you make using counters?
  What do you notice?
  Are there any patterns?

Mathematical Talk

Why are square numbers called ‘square’ numbers?

Is there a pattern between the numbers?

True or False: The square of an even number is even and the square of an odd number is odd.
### Square Numbers

#### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Teddy says,</th>
<th>Factors come in pairs so all whole numbers must have an even number of factors.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you agree? Explain your reasoning.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Children will find that some numbers don’t have an even number of factors. e.g. 25 Square numbers have an odd number of factors.</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Julian thinks that $4^2$ is equal to 16.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you agree? Convince me.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>He also thinks that $6^3$ is equal to 12.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you agree? Explain what you have noticed.</td>
</tr>
</tbody>
</table>

| Children may use concrete materials or draw pictures to prove it. |

| Children should spot that 6 has been multiplied by 2. They may create the array to prove that $6^2 = 36$ and $6 \times 2 = 12$. |

### How many square numbers can you make by adding prime numbers together?

<table>
<thead>
<tr>
<th>Here’s one to get you started:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 + 2 = 4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solutions include:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 + 2 = 4$</td>
</tr>
<tr>
<td>$2 + 7 = 9$</td>
</tr>
<tr>
<td>$11 + 5 = 16$</td>
</tr>
<tr>
<td>$23 + 2 = 25$</td>
</tr>
<tr>
<td>$29 + 7 = 36$</td>
</tr>
</tbody>
</table>

### Always, sometimes, never.

A square number has an even number of factors.

| Never. Square numbers have an odd number of factors. |
Cube Numbers

Notes and Guidance

Children learn that a cubed number is the product of three numbers which are the same.

If you multiply a number by itself, then itself again the result is a cubed number.

They learn the notation for cubed is \( \times^3 \).

Mathematical Talk

Why are cube numbers called ‘cube’ numbers?

How are squared and cubed numbers similar?

How are they different?

True or False: Cubes of even numbers are even and cubes of odd numbers are odd.

Varied Fluency

Use multilink cubes and investigate how many are needed to make different sized cubes.

How many multilink cubes are required to make the first cube number? The second? Third?

Can you predict what the tenth cube number is going to be?

Complete the table.

<table>
<thead>
<tr>
<th></th>
<th>(3^3)</th>
<th>(5^3)</th>
<th>(4^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(3 \times 3 \times 3)</td>
<td>(5 \times 5 \times 5)</td>
<td>(6 \times 6 \times 6)</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td></td>
<td>8</td>
</tr>
</tbody>
</table>

Calculate:

\[4^3 = \quad \quad 5^3 = \quad \quad 4 \text{ cubed} = \quad \quad 6 \text{ cubed} = \quad \quad\]
### Cube Numbers

#### Reasoning and Problem Solving

**Rosie says,**

$$5^3$$ is equal to 15

**Rosie is wrong,** she has multiplied 5 by 3 rather than by itself 3 times.

**Do you agree? Explain your answer.**

Here are 3 number cards

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
</table>

Each number card is a cubed number. Use the information to find each number.

- $$A \times A = B$$
- $$B + B - 3 = C$$
- Digit total of $$C = A$$

A = 8
B = 64
C = 125

Jenny is thinking of a two-digit number that is both a square and a cube number. What number is she thinking of?

Caroline’s daughter has an age that is a cube number. Next year her age will be a square number. How old is she now?

The sum of a cube number and a square number is 150. What are the two numbers?

<table>
<thead>
<tr>
<th>64</th>
<th>8 years old</th>
</tr>
</thead>
<tbody>
<tr>
<td>125 &amp; 25</td>
<td></td>
</tr>
</tbody>
</table>
**Multiply by 10, 100 & 1,000**

**Notes and Guidance**

Children recap multiplying by 10 and 100 before moving on to multiplying by 1,000.

They look at numbers in a place value grid and discuss how many places to the left digits move when you multiply by different multiples of 10.

**Mathematical Talk**

Which direction do the digits move when you multiply by 10, 100 or 1,000?

How many places do you move to the left?

When we have an empty place value column to the right of our digits what number do we use as a place holder?

Can you use multiplying by 100 to help you multiply by 1,000? Explain why.

**Varied Fluency**

Make the number 234 on a place value grid using counters.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>HTh</td>
<td>TTh</td>
<td>Th</td>
<td>H</td>
<td>T</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When I multiply my number by 10, where will I move my counters?
Is this always the case?

Complete the following questions using counters and a place value grid.

\[
234 \times 100 = ____ \\
100 \times 36 = ____ \\
45,020 \times 10 = ____ \\
324 \times 100 = ____ \\
1,000 \times 207 = ____ \\
____ = 3,456 \times 1,000
\]

Use <, > or = to complete the statements.

\[
62 \times 1,000 \quad \square \quad 62 \times 100 \\
100 \times 32 \quad \square \quad 32 \times 100 \\
48 \times 100 \quad \square \quad 48 \times 10 \times 10 \times 10
\]
### Multiply by 10, 100 & 1,000

**Reasoning and Problem Solving**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rosie has £300 in her bank account.</td>
<td></td>
</tr>
<tr>
<td>Louis has 100 times more than Rosie in his bank account.</td>
<td></td>
</tr>
<tr>
<td>How much more money does Louis have than Rosie?</td>
<td></td>
</tr>
<tr>
<td>Rosie has £300</td>
<td></td>
</tr>
<tr>
<td>Louis has £30,000</td>
<td></td>
</tr>
<tr>
<td>Louis has £29,700 more than Rosie.</td>
<td></td>
</tr>
<tr>
<td>Emily has £1,020 in her bank account.</td>
<td></td>
</tr>
<tr>
<td>Philip has £120 in his bank account.</td>
<td></td>
</tr>
<tr>
<td>Emily says, ‘I have ten times more money than you.’</td>
<td></td>
</tr>
<tr>
<td>Emily is incorrect, she would have £1,200 if this was the case.</td>
<td></td>
</tr>
<tr>
<td>Jack is thinking of a 3-digit number.</td>
<td></td>
</tr>
<tr>
<td>When he multiplies his number by 100, the ten thousands and hundreds digit are the same.</td>
<td>181</td>
</tr>
<tr>
<td>The sum of the digits is 10</td>
<td></td>
</tr>
<tr>
<td>What number could Jack be thinking of?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>262</td>
</tr>
<tr>
<td></td>
<td>343</td>
</tr>
<tr>
<td></td>
<td>424</td>
</tr>
<tr>
<td></td>
<td>505</td>
</tr>
</tbody>
</table>

Is Emily correct? Explain your reasoning.
Divide by 10, 100 & 1,000

Notes and Guidance

Children look at dividing by 10, 100 and 1,000 using a place value chart.

They use counters and digits to learn that the digits move to the right when dividing by powers of ten.

Varied Fluency

What number is represented in the place value grid?
Divide the number by 100
Which direction do the counters move?
How many columns do they move?
What number do we have now?

Complete the following using a place value grid.
• Divide 460 by 10
• Divide 5,300 by 100
• Divide 62,000 by 1,000

Divide these numbers by 10, 100 and 1,000

80,000 300,000 547,000

Calculate 45,000 ÷ 10 ÷ 10
How else could you write this?

Mathematical Talk

What happens to the digits?
How are dividing by 10, 100 and 1,000 related to each other?
How are dividing by 10, 100 and 1,000 linked to multiplying by 10, 100 and 1,000?
What does ‘inverse’ mean?
Divide by 10, 100 & 1,000

Reasoning and Problem Solving

David has £357,000 in his bank.
He divides the amount by 1,000 and takes that much money out of the bank.
Using the money he has taken out, he buys some furniture costing two hundred and sixty-nine pounds.
How much money does David have left from the money he took out?
Show your working out.

\[
\begin{align*}
357,000 \div 1,000 &= 357 \\
\text{If you subtract £269, he is left with £88}
\end{align*}
\]

Here are the answers to some problems:

\[
\begin{align*}
5,700 & \\
405 & \\
397 & \\
6,203 & 
\end{align*}
\]

Possible solutions could be:

\[
\begin{align*}
3,970 \div 10 &= 397 \\
57,000 \div 10 &= 5,700 \\
397,000 \div 1,000 &= 397 \\
40,500 \div 100 &= 405 \\
620,300 \div 100 &= 6,203
\end{align*}
\]

Can you write at least two questions for each answer involving dividing by 10, 100 or 1,000?
Multiples of 10, 100 & 1,000

Notes and Guidance

Children have been taught how to multiply and divide by 10, 100 and 1,000.
They now use knowledge of other multiples to calculate related questions.

Mathematical Talk

If we are multiplying by 20, can we break it down into two steps and use our knowledge of multiplying by 10?

How does using multiplication and division as inverses help us use known facts?

Varied Fluency

36 × 5 = 180
Use this fact to solve the following questions:

36 × 50 = ___  500 × 36 = ___
5 × 360 = ___  360 × 500 = ___

Here are two methods to solve 24 × 20

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 × 10 × 2</td>
<td>24 × 2 × 10</td>
</tr>
<tr>
<td>= 240 × 2</td>
<td>= 48 × 10</td>
</tr>
<tr>
<td>= 480</td>
<td>= 480</td>
</tr>
</tbody>
</table>

What is the same about the methods, what is different?

Use the division diagram to help solve the calculations.

7,200 ÷ 200 = 36

3,600 ÷ 200 = ___
18,000 ÷ 200 = ___
5,400 ÷ ___ = 27
___ = 6,600 ÷ 200
Tim has answered a question.
Here is his working out.

\[
\begin{align*}
600 \div 25 &= 300 \\
600 \div 2 &= 300 \\
300 \div 5 &= 60 \\
600 \div 25 &= 60
\end{align*}
\]

Is he correct?
Explain your answer.

Tim is not correct as he has partitioned 25 incorrectly.

He could have divided by 5 twice.

The correct answer should be 24

60 \times 7 = 420 \quad 420 \div 70 = ____

Alex is wrong because
\[60 \times 70 = 4200\]
and
\[6 \times 70 = 420\]

The answer is 60 because all of the numbers are 10 times bigger.

Do you agree with Alex?
Explain your answer.