Autumn Scheme of Learning

Year 6

#MathsEveryoneCan

2019-20
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Overview

Small Steps

- Simplify fractions
- Fractions on a number line
- Compare and order (denominator)
- Compare and order (numerator)
- Add and subtract fractions (1)
- Add and subtract fractions (2)
- Add fractions
- Subtract fractions
- Mixed addition and subtraction
- Multiply fractions by integers
- Multiply fractions by fractions
- Divide fractions by integers (1)
- Divide fractions by integers (2)
- Four rules with fractions
- Fraction of an amount
- Fraction of an amount – find the whole

NC Objectives

Use common factors to simplify fractions; use common multiples to express fractions in the same denomination.

Compare and order fractions, including fractions > 1

Generate and describe linear number sequences (with fractions)

Add and subtract fractions with different denominations and mixed numbers, using the concept of equivalent fractions.

Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]

Divide proper fractions by whole numbers [for example $\frac{1}{3} \div 2 = \frac{1}{6}$]

Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example $\frac{1}{8}$]

Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.
Simplify Fractions

Notes and Guidance

Children use their understanding of the highest common factor to simplify fractions, building on their knowledge of equivalent fractions in earlier years.

Children apply their understanding when calculating with fractions and simplifying their answers. Encourage children to use pictorial representations to support simplifying e.g. a fraction wall.

Mathematical Talk

Can you make a list of the factors for each number? Which numbers appear in both lists? What do we call these (common factors)?

What is the highest common factor of the numerator and denominator?

Is a simplified fraction always equivalent to the original fraction? Why?

If the HCF of the numerator and denominator is 1, can it be simplified?

Varied Fluency

Alex is simplifying $\frac{8}{12}$ by dividing the numerator and denominator by their highest common factor.

Factors of 8: 1, 2, 4, 8

Factors of 12: 1, 2, 3, 4, 6, 12

4 is the highest common factor.

Use Alex’s method to simplify these fractions:

$\frac{6}{9}, \frac{6}{18}, \frac{10}{18}, \frac{10}{15}, \frac{15}{50}$

Mo has 3 boxes of chocolates. 2 boxes are full and one box is $\frac{4}{10}$ full.

To simplify $2 \frac{4}{10}$, keep the whole number the same and simplify the fraction. $\frac{4}{10}$ simplifies to $\frac{2}{5}$.

$2 \frac{4}{10} = 2 \frac{2}{5}$

Use Mo’s method to simplify:

$\frac{3}{8}, \frac{4}{21}, \frac{5}{21}, \frac{9}{10}, \frac{2}{6}, \frac{7}{10}, \frac{32}{10}, \frac{32}{6}$
Year 6 | Autumn Term | Week 7 to 10 – Number: Fractions

Simplify Fractions

Reasoning and Problem Solving

Find the total of the fractions.
Give your answer in its simplest form.

\[
\frac{5}{9} + \frac{1}{9} = \frac{5}{9} + \frac{3}{9} = \frac{5}{9} + \frac{7}{9} =
\]

Do all the answers need simplifying?
Explain why.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Simplifies to (\frac{1}{2})</th>
<th>Simplifies to (\frac{1}{3})</th>
<th>Simplifies to (\frac{1}{4})</th>
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<tbody>
<tr>
<td>(\frac{5}{15})</td>
<td>(\frac{2}{4})</td>
<td>(\frac{4}{16})</td>
<td>(\frac{8}{16})</td>
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<tr>
<td>(\frac{5}{10})</td>
<td>(\frac{3}{9})</td>
<td>(\frac{6}{12})</td>
<td>(\frac{2}{8})</td>
</tr>
</tbody>
</table>

Sort the fractions into the table.

Tommy is simplifying \(4 \frac{12}{16}\)

\[
4 \frac{12}{16} = 1 \frac{3}{4}
\]

Explain Tommy’s mistake.

Tommy has divided the whole number by 4 instead of just simplifying \(\frac{12}{16}\) by dividing the numerator and denominator by 4.

Can you see any patterns between the numbers in each column?
What is the relationship between the numerators and denominators?
Can you add three more fractions to each column?

Complete the sentence to describe the patterns:
When a fraction is equivalent to \(\frac{1}{2}\), the numerator is _______ the denominator.

Repeat for \(\frac{1}{3}\) and \(\frac{1}{4}\).
Fractions on a Number Line

Notes and Guidance

Children count forwards and backwards in fractions. They compare and order fractions with the same denominator or denominators that are multiples of the same number. Encourage children to draw extra intervals on the number lines to support them to place the fractions more accurately. Children use the divisions on the number line to support them in finding the difference between fractions.

Mathematical Talk

Which numbers do I say when I count in eighths and when I count in quarters?

Can you estimate where the fractions will be on the number line?

Can you divide the number line into more intervals to place the fractions more accurately?

How can you find the difference between the fractions?

Varied Fluency

Jack is counting in quarters. He writes each number he says on a number line. Complete Jack’s number line.

Can you simplify any of the fractions on the number line? Can you count forward in eighths? How would the number line change?

Place \(\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{5}{8}, \frac{7}{8}\) and \(\frac{3}{16}\) on the number line.

Which fractions were the easiest to place? Which fractions were the hardest to place? Which fraction is the largest? Which fraction is the smallest? What is the difference between the largest and smallest fraction?
Rosie is counting backwards in fifths.
She starts at $\frac{3}{2}$ and counts back nine fifths.
What number does Rosie end on?
Show this on a number line.

Rosie ends on $1 \frac{3}{5}$

Plot the sequences on a number line.

$3 \frac{1}{2}, 4, 4 \frac{1}{2}, 5, 5 \frac{1}{2}, 6$

$\frac{13}{4}, \frac{15}{4}, \frac{17}{4}, \frac{19}{4}, \frac{21}{4}, \frac{23}{4}$

$5 \frac{5}{8}, 5 \frac{1}{8}, 4 \frac{5}{8}, 4 \frac{1}{8}, 3 \frac{5}{8}, 3 \frac{1}{8}$

Various answers available.

How many ways can you show a difference of one quarter on the number line?

0 ||||| 1 ||||| 2

Which sequence is the odd one out?
Explain why.

Can you think of a reason why each of the sequences could be the odd one out?

Children may choose different sequences for different reasons. First sequence: the only one containing 6 or it is the only one containing whole numbers.
Second sequence: only one using improper fractions.
Third sequence: the only one going backwards.
Fourth sequence: only one not counting in halves.
Compare & Order (Denominator)

Notes and Guidance

Children use their knowledge of equivalent fractions to compare fractions where the denominators are not multiples of the same number.
They find the lowest common multiple of the denominators in order to find equivalent fractions with the same denominators.
Children then compare the numerators to find the larger or smaller fraction. Encourage children to also use their number sense to visualise the size of the fractions before converting.

Mathematical Talk

When I know the lowest common multiple, how do I know what to multiply the numerator and denominator by to find the correct equivalent fraction?
How is comparing mixed numbers different to comparing proper fractions? Do I need to compare the whole numbers?
Why? If the whole numbers are the same, what do I do?
Can you plot the fractions on a number line to estimate which is the smallest? Which fractions are larger/smaller than a half?
How does this help me order the fractions?

Varied Fluency

Use the bar models to compare $\frac{3}{4}$ and $\frac{2}{3}$.
___ is greater than ___
___ is less than ___

Dora is comparing $\frac{5}{6}$ and $\frac{3}{4}$ by finding the lowest common multiple of the denominators.
Multiples of 6: 6, 12, 18, 24
Multiples of 4: 4, 8, 12, 16,
12 is the LCM of 4 and 6

$\frac{5}{6} = \frac{10}{12} \quad \frac{3}{4} = \frac{9}{12}$

$\frac{10}{12} > \frac{9}{12}$

Use Dora’s method to compare the fractions.

$\frac{4}{5} \quad \frac{3}{4} \quad \frac{3}{5} \quad \frac{4}{7} \quad \frac{3}{4} \quad \frac{7}{10} \quad 2\frac{2}{5} \quad 2\frac{3}{8}$

Order the fractions in descending order.

\[
\frac{3}{8}, \frac{11}{20}, \frac{7}{5}, \frac{4}{10}, \frac{2}{3}, \frac{3}{7}
\]

Which fraction is the greatest?
Which fraction is the smallest?
**Compare & Order (Denominator)**

**Reasoning and Problem Solving**

Use the digit cards to complete the statements.

\[
\begin{align*}
\frac{5}{4} &> \frac{6}{3} < \frac{3}{4} < \frac{6}{4} < \frac{6}{3}
\end{align*}
\]

Find three examples of ways you could complete the statement.

\[
\begin{align*}
\frac{3}{5} &< \frac{6}{4} \\
\frac{3}{4} &< \frac{6}{5} \\
\frac{4}{5} &< \frac{6}{3}
\end{align*}
\]

Teddy is comparing \(\frac{3}{8}\) and \(\frac{5}{12}\).

Teddy is incorrect because the LCM of 8 and 12 is 24. 96 is a common multiple so he would still compare the fractions correctly but it is not the most efficient method.

To find the lowest common multiple, I will multiply 8 and 12 together, \(8 \times 12 = 96\). I will use a common denominator of 96.

Is Teddy correct? Explain why.

Can one of your ways include an improper fraction?

More answers available.
Compare & Order (Numerator)

Notes and Guidance

Building on their prior knowledge of comparing unit fractions, children look at comparing fractions by finding a common numerator. They focus on the idea that when the numerators are the same, the larger the denominator, the smaller the fraction.

Children consider the most efficient method when comparing fractions and decide whether to find common numerators or common denominators.

Mathematical Talk

What’s the same and what’s different about the fractions on the bar models? How can we compare them? Can you use the words greatest and smallest to complete the sentences?

Do you need to change one or both numerators? Why?

How can you decide whether to find a common numerator or denominator?

Varied Fluency

Compare the fractions.

When the denominators are the same, the ______ the numerator, the ______ the fraction.

When the numerators are the same, the ______ the denominator, the ______ the fraction.

Jack is comparing $\frac{2}{5}$ and $\frac{4}{7}$ by finding the LCM of the numerators.

The LCM of 2 and 4 is 4

Use Jack’s method to compare the fractions.

$\frac{2}{5} = \frac{4}{10}$ and $\frac{4}{10} < \frac{4}{7}$
Mo is comparing the fractions $\frac{3}{7}$ and $\frac{6}{11}$.

He wants to find a common denominator.

Explain whether you think this is the most effective strategy.

This is not the most effective strategy because both denominators are prime. He could find a common numerator by changing $\frac{3}{7}$ into $\frac{6}{14}$ and comparing them by using the rule ‘when the numerator is the same, the smaller the denominator, the bigger the fraction’ $\frac{6}{11}$ is bigger.

Two different pieces of wood have had a fraction chopped off.

Here are the pieces now, with the fraction that is left.

\[
\begin{align*}
\text{5} & \quad \text{6} \\
\text{3} & \quad \text{4}
\end{align*}
\]

Which piece of wood was the longest to begin with?

Explain your answer.

Can you explain your method?

The second piece was longer because $\frac{1}{4}$ is greater than $\frac{1}{6}$. Children can explain their methods and how they compared one quarter and one sixth.
Add & Subtract Fractions (1)

Notes and Guidance

Children add and subtract fractions within 1 where the denominators are multiples of the same number. Encourage children to find the lowest common multiple in order to find a common denominator. Ensure children are confident with the understanding of adding and subtracting fractions with the same denominator. Bar models can support this, showing children that the denominators stay the same whilst the numerators are added or subtracted.

Mathematical Talk

If the denominators are different, when we are adding or subtracting fractions, what do we need to do? Why?

How does finding the lowest common multiple help to find a common denominator?

Can you use a bar model to represent Eva’s tin of paint? On which day did Eva use the most paint? On which day did Eva use the least paint? How much more paint did Eva use on Friday than Saturday?

Varied Fluency

Whitney is calculating $\frac{5}{8} + \frac{3}{16}$.
She finds the lowest common multiple of 8 and 16 to find a common denominator.

LCM of 8 and 16 is 16

\[
\frac{5}{8} = \frac{10}{16} \quad \frac{10}{16} + \frac{3}{16} = \frac{13}{16}
\]

Use this method to calculate:

\[
\frac{1}{3} + \frac{2}{9} = \frac{3}{7} + \frac{7}{21} = \frac{8}{15} + \frac{1}{5} = \frac{3}{16} + \frac{3}{8} + \frac{1}{4} =
\]

Find a common denominator for each pair of fractions by using the lowest common multiple. Subtract the smaller fraction from the larger fraction in each pair.

\[
\frac{3}{4} - \frac{5}{8} \quad \frac{7}{12} - \frac{1}{3} \quad \frac{11}{16} - \frac{3}{4} \quad \frac{14}{15} - \frac{2}{5} \quad \frac{8}{9} - \frac{1}{3}
\]

Eva has a full tin of paint. She uses $\frac{1}{3}$ of the tin on Friday, $\frac{1}{21}$ on Saturday and $\frac{2}{7}$ on Sunday. How much paint does she have left?
Add & Subtract Fractions (1)

Reasoning and Problem Solving

Use the same digit in both boxes to complete the calculation.
Is there more than one way to do it?

\[ \frac{4}{20} + \frac{1}{5} = \frac{9}{20} \]

Alex is adding fractions.

\[ \frac{3}{5} + \frac{1}{15} = \frac{4}{20} = \frac{1}{5} \]

Do you agree with her?
Explain your answer.

Alex is wrong because she has added the numerators and the denominators rather than finding a common denominator. It should be

\[ \frac{9}{15} + \frac{1}{15} = \frac{10}{15} = \frac{2}{3} \]

Dexter subtracted \( \frac{3}{5} \) from a fraction and his answer was \( \frac{8}{45} \).
What fraction did he subtract \( \frac{3}{5} \) from?
Give your answer in its simplest form.

\[ \frac{8}{45} + \frac{3}{5} = \frac{8}{45} + \frac{27}{45} \]

\[ \frac{8}{45} + \frac{27}{45} = \frac{35}{45} = \frac{7}{9} \]

Dexter subtracted \( \frac{3}{5} \) from \( \frac{7}{9} \).
Add & Subtract Fractions (2)

Notes and Guidance

Children add and subtract fractions where the denominators are not multiples of the same number. They continue to find the lowest common multiple, but now need to find equivalent fractions for both fractions in the calculation to find a common denominator.

When the denominators are not multiples of the same number, support children to notice that we need to multiply the denominators together in order to find the LCM.

Mathematical Talk

What is the same about all the subtractions? \( \frac{3}{4} \)

What do you notice about the LCM of all the denominators?

Which of the subtractions has the biggest difference? Explain how you know. Can you order the differences in ascending order?

How can we find the LCM of three numbers? Do we multiply them together? Is 120 the LCM of 4, 5 and 6?

Varied Fluency

- Amir is calculating \( \frac{7}{9} - \frac{1}{2} \).
  He finds the lowest common multiple of 9 and 2
  LCM of 9 and 2 is 18
  \[ \frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18} \]

  Use this method to calculate:
  \[ \frac{3}{4} - \frac{1}{3} = \frac{3}{4} - \frac{3}{5} = \frac{3}{4} - \frac{2}{7} = \frac{3}{4} - \frac{7}{11} = \]

- Eva has a bag of carrots weighing \( \frac{3}{4} \) kg and a bag of potatoes weighing \( \frac{2}{5} \) kg. She is calculating how much they weigh altogether.

  The LCM of 4 and 5 is 20. I will convert the fractions to twentieths.
  \[ \frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1 \frac{3}{20} \text{ kg} \]

  Use this method to calculate:
  \[ \frac{1}{4} + \frac{2}{5} = \frac{7}{8} + \frac{1}{3} = \frac{5}{6} + \frac{5}{7} = \frac{13}{20} + \frac{2}{3} = \]

- On Friday, Ron walks \( \frac{5}{6} \) km to school, \( \frac{3}{4} \) km to the shops and \( \frac{4}{5} \) km home. How far does he walk altogether?
Add & Subtract Fractions (2)

Reasoning and Problem Solving

A car is travelling from Halifax to Brighton.
In the morning, it completes \(\frac{2}{3}\) of the journey.
In the afternoon, it completes \(\frac{1}{5}\) of the journey.
What fraction of the journey has been travelled altogether?
What fraction of the journey is left to travel?

If the journey is 270 miles, how far did the car travel in the morning?
How far did the car travel in the afternoon?
How far does the car have left to travel?

The car has travelled \(\frac{13}{15}\) of the journey altogether.
There is \(\frac{2}{15}\) of the journey left to travel.

Mr and Mrs Rose and knitting scarves.
Mr Rose's scarf is \(\frac{5}{9}\) m long.
Mrs Rose's scarf is \(\frac{1}{5}\) m longer than Mr Rose's scarf.
How long is Mrs Rose's scarf?
How long are both the scarves altogether?

Mrs Rose's scarf is \(\frac{34}{45}\) m long.
Both scarves together are \(1\frac{14}{45}\) m long.

Fill in the boxes to make the calculation correct.

The car travelled 180 miles in the morning.
The car travelled 54 miles in the afternoon.
The car has 36 miles left to travel.

Various answers available. E.g.

\[
\begin{array}{c}
1 \frac{1}{10} = \frac{3}{5} + \frac{5}{10}
\end{array}
\]
Add Fractions

Notes and Guidance

Children explore adding mixed numbers. They look at different methods depending on whether the fractions total more than one. They add fractions with any denominators, building on their understanding from the previous steps. Encourage children to draw bar models to support them in considering whether the fractions will cross the whole. They continue to simplify answers and convert between improper fractions and whole numbers when calculating.

Mathematical Talk

How many wholes are there altogether?
Can you find the LCM of the denominators to find a common denominator?
Do you prefer Tommy or Whitney's method? Why?
Does Tommy's method work when the fractions add to more than one? How could we adapt his method?
Does Whitney's method work effectively when there are large whole numbers?

Varied Fluency

- Tommy is adding mixed numbers. He adds the wholes and then adds the fractions. Then, Tommy simplifies his answer.

\[1 \frac{1}{2} + 2 \frac{1}{6} = 1 \frac{3}{6} + 2 \frac{1}{6} = 3 \frac{4}{6} = 3 \frac{2}{3}\]

Use Tommy's method to add the fractions.
\[3 \frac{1}{2} + 2 \frac{3}{8} = \quad 34 \frac{1}{9} + 5 \frac{2}{5} = \quad 12 \frac{5}{12} + 2 \frac{1}{7} = \]

- Whitney is also adding mixed numbers. She converts them to improper fractions, adds them, and then converts them back to a mixed number.

\[1 \frac{1}{2} + 2 \frac{1}{6} = \frac{3}{2} + \frac{13}{6} = \frac{9}{6} + \frac{13}{6} = \frac{22}{6} = 3 \frac{4}{6} = 3 \frac{2}{3}\]

Use Whitney's method to add the fractions.
\[3 \frac{1}{2} + 2 \frac{3}{8} \quad 2 \frac{1}{9} + 2 \frac{2}{5} \quad 2 \frac{7}{9} + 2 \frac{2}{5} \quad 4 \frac{3}{4} + 3 \frac{11}{15}\]

- Jug A has \(2 \frac{3}{4}\) litres of juice in it. Jug B has \(3 \frac{4}{5}\) litres of juice in it. How much juice is there in Jug A and Jug B altogether?
Add Fractions

Reasoning and Problem Solving

Each row and column adds up to make the total at the end.
Use this information to complete the diagram.

\[
\begin{array}{ccc}
2 & \frac{1}{4} & 1 \frac{1}{2} \\
\frac{1}{8} & & \\
3 & \frac{1}{12} & \\
\hline
5 \frac{1}{2} & & \\
\end{array}
\]

\[= 3 \frac{7}{8}\]

Dora is baking muffins.
She uses \(2 \frac{1}{2}\) kg of flour, \(1 \frac{3}{5}\) kg of sugar and \(1 \frac{1}{4}\) kg of butter.

How much flour, sugar and butter does she use altogether?

How much more flour does she use than butter?

How much less butter does she use than sugar?

Dora uses \(5 \frac{7}{20}\) kg of flour, sugar and butter altogether.

Dora uses \(1 \frac{1}{4}\) kg more flour than butter.

Dora uses \(\frac{7}{20}\) kg less butter than sugar.
Subtract Fractions

Notes and Guidance

Children subtract mixed numbers. They explore different methods including exchanging wholes for fractions and subtracting the wholes and fractions separately and converting the mixed number to an improper fraction. Encourage children to consider which method is the most efficient depending on the fractions they are subtracting. Bar models can support to help children to visualise the subtraction and understand the procedure.

Mathematical Talk

How many eighths can we exchange for one whole?

What is the same about the first set of subtractions?

What is different about the subtractions? (How does this affect the subtraction?)

Do you prefer Annie’s or Amir’s method? Why?

Look at Amir’s calculation, what do you notice about the relationship between $3 \frac{2}{5}$ and $1 \frac{7}{10}$? ($3 \frac{2}{5}$ is double $1 \frac{7}{10}$)

Varied Fluency

- Annie is calculating $3 \frac{1}{4} - 1 \frac{3}{4}$

  I can’t subtract the wholes and fractions separately because $\frac{1}{4}$ is less than $\frac{3}{4}$. I will exchange 1 whole for 4 quarters. $3 \frac{1}{4} = 2 \frac{5}{4}$

  $3 \frac{1}{4} - 1 \frac{3}{4} = 2 \frac{5}{4} - 1 \frac{3}{4} = 1 \frac{2}{4} = 1 \frac{1}{2}$

- Use Annie’s method to calculate:
  
  $3 \frac{1}{8} - 1 \frac{3}{8}$
  $3 \frac{1}{8} - 1 \frac{1}{2}$
  $3 \frac{1}{8} - 1 \frac{1}{5}$
  $3 \frac{1}{8} - 1 \frac{3}{5}$

- Amir is calculating $3 \frac{2}{5} - 1 \frac{7}{10}$

  He converts the mixed numbers to improper fractions to subtract them.

  $3 \frac{2}{5} - 1 \frac{7}{10} = \frac{17}{5} - \frac{17}{10} = \frac{34}{10} - \frac{17}{10} = \frac{17}{10} = 1 \frac{7}{10}$

- Convert the mixed numbers to improper fractions to calculate:

  $4 \frac{4}{5} - 1 \frac{9}{10}$
  $2 \frac{1}{7} - 1 \frac{1}{3}$
  $3 \frac{5}{12} - 1 \frac{7}{9}$
  $3 \frac{5}{11} - 1 \frac{4}{5}$
Subtract Fractions

Reasoning and Problem Solving

A blue, orange and green box are on a number line.

1 \( \frac{1}{16} \) 3

The number in the green box is \( 3 \frac{2}{3} \) more than the orange box.

The number in the orange box is:

The number in the orange box is \( \frac{11}{16} \) greater than the number in the blue box.

5 \( \frac{5}{12} \) – 3 \( \frac{2}{3} \) = 1 \( \frac{9}{12} \)

The orange box is

1 \( \frac{3}{4} \)

1 \( \frac{3}{4} \) – 1 \( \frac{1}{16} \) = \( \frac{11}{16} \)

The orange box is \( \frac{11}{16} \) greater than the blue box.

Complete the part-whole model.

2 \( \frac{1}{6} \) – 1 \( \frac{4}{5} \) = 1 \( \frac{11}{30} \)

1 \( \frac{4}{5} \) – \( \frac{8}{10} \) = 1

1 \( \frac{11}{30} \) – \( \frac{5}{30} \) = 1 \( \frac{6}{30} \) = 1 \( \frac{1}{5} \)

Jack is calculating 4 \( \frac{2}{7} \) – 2 \( \frac{6}{7} \)

He adds \( \frac{1}{7} \) to both numbers.

4 \( \frac{2}{7} \) – 2 \( \frac{6}{7} \) = 4 \( \frac{3}{7} \) – 3

so the answer is 1 \( \frac{3}{7} \)

Explain why Jack is correct.

Jack has increased both mixed numbers by \( \frac{1}{7} \) so the difference has remained constant.
**Mixed Addition & Subtraction**

**Notes and Guidance**

Children solve problems that involve adding and subtracting fractions and mixed numbers. Encourage children to consider the most efficient method of adding and subtracting fractions and to simplify their answers when possible. Children can use bar models to represent the problems and support them in deciding whether they need to add or subtract. They can share their different methods to gain a flexible approach to calculating with fractions.

**Mathematical Talk**

Can you draw a bar model to represent the problem? Do we need to add or subtract the fractions?

How do I know if my answer is simplified fully?

What is the lowest common multiple of the denominators?

How can I calculate the area covered by each vegetable? If you know the area for carrots and cabbages, how can you work out the area for potatoes? Can you think of 2 different ways?

**Varied Fluency**

- **Alex has 5 bags of sweets.**
  On Monday she eats \(\frac{2}{3}\) of a bag and gives \(\frac{4}{5}\) of a bag to her friend.
  On Tuesday she eats \(1\frac{1}{3}\) bags and gives \(\frac{2}{5}\) of a bag to her friend.
  What fraction of her sweets does Alex have left?
  Give your answer in its simplest form.

- **Here is a vegetable patch.** \(\frac{1}{5}\) of the patch is for carrots. \(\frac{3}{8}\) of the patch is for cabbages.
  What fraction of the patch is for carrots and cabbages altogether?
  What fraction of the patch is for potatoes?
  What fraction more of the patch is for potatoes than cabbages?
  Give your answers in their simplest form.

The vegetable patch has an area of 80 m\(^2\)
What is the area covered by each vegetable?
The mass of Annie’s suitcase is $29\frac{1}{2}$ kg.
Teddy’s suitcase is $2\frac{1}{5}$ kg lighter than Annie’s.
How much does Teddy’s suitcase weigh?
How much do the suitcases weigh altogether?

There is a weight allowance of 32 kg per suitcase.
How much below the weight allowance are Annie and Teddy?

Teddy’s suitcase weighs $27\frac{3}{10}$ kg
The suitcases weigh $56\frac{4}{5}$ kg altogether.

Annie is $2\frac{1}{2}$ kg under the weight allowance.
Teddy is $4\frac{7}{10}$ kg under the weight allowance.

Find the value of the $\heartsuit + 3\frac{4}{9} = 6\frac{1}{3}$
$8\frac{1}{10} - \heartsuit = \odot$

The value of the $\odot$ is $2\frac{8}{9}$
The value of the $\heartsuit$ is $5\frac{19}{90}$
Multiply Fractions by Integers

Notes and Guidance

Children multiply fractions and mixed numbers by integers. They use diagrams to highlight the link between multiplication and repeated addition. This supports the children in understanding why the denominator stays the same and we multiply the numerator.

When multiplying mixed numbers, children partition into wholes and parts to multiply more efficiently. They compare this method with multiplying improper fractions.

Mathematical Talk

How is multiplying fractions similar to adding fractions?

How does partitioning the mixed number into wholes and fractions support us to multiply?

Do you prefer partitioning the mixed number or converting it to an improper fraction to multiply? Why?

Does it matter if the integer is first or second in the multiplication sentence? Why?

Varied Fluency

Complete:

\[ 3 \times \frac{2}{3} \]

\[ 4 \times \frac{7}{8} \]

\[ \frac{2}{5} \times 7 \]

\[ \frac{3}{5} \times \frac{2}{5} \]

Eva partitions \( 2\frac{3}{5} \) to help her to calculate \( 2\frac{3}{5} \times 3 \)

\[ 2 \times 3 = 6 \]

\[ \frac{3}{5} \times 3 = \frac{9}{5} = 1\frac{4}{5} \]

\[ 6 + 1\frac{4}{5} = 7\frac{4}{5} \]

Use Eva's method to calculate:

\[ 2\frac{5}{6} \times 3 \quad 1\frac{3}{7} \times 5 \quad 2\frac{2}{3} \times 3 \quad 4 \times 1\frac{1}{6} \]

Convert the mixed number to an improper fraction to multiply.

\[ 2\frac{3}{5} \times 3 = \frac{13}{5} \times 3 = \frac{39}{5} = 7\frac{4}{5} \]

Use this method to calculate:

\[ 3 \times 2\frac{2}{5} \quad 1\frac{5}{7} \times 3 \quad 2 \times 1\frac{3}{4} \quad 2 \times 1\frac{1}{6} \]
Multiply Fractions by Integers

Reasoning and Problem Solving

There are 9 lamp posts on a road. There is $4 \frac{3}{8}$ of a metre between each lamp post.

What is the distance between the first and last lamp post?

Use pattern blocks, if possible, to work out what fraction the other shapes represent.

Use this to calculate the multiplications. Give your answers in their simplest form.

\[
\begin{align*}
\triangle \times 5 &= \frac{5}{6} \\
\square \times 5 &= \frac{5}{3} = 1 \frac{2}{3} \\
\square \times 5 &= \frac{5}{2} = 2 \frac{1}{2}
\end{align*}
\]

Eva and Amir both work on a homework project.

I spent $4 \frac{1}{4}$ hours a week for 4 weeks doing my project.

I spent $2 \frac{3}{4}$ hours a week for 5 weeks doing my project.

Who spent the most time on their project?

Explain your reasoning.

\[
\begin{align*}
8 \times 4 \frac{3}{8} &= 8 \times \frac{35}{8} \\
&= \frac{280}{8} = 35 \\
\text{The distance between the first and last lamp post is 35 metres.}
\end{align*}
\]

\[
\begin{align*}
4 \times 4 \frac{1}{4} &= \frac{68}{4} \\
&= 17 \text{ hours} \\
5 \times 2 \frac{3}{4} &= \frac{55}{4} \\
&= 13 \frac{3}{4} \text{ hours}
\end{align*}
\]

Eva spent $3 \frac{1}{4}$ hours longer on her project than Amir did.
Multiply Fractions by Fractions

Notes and Guidance

Children use concrete and pictorial representations to support them to multiply fractions. Support children in understanding the link between multiplying fractions and finding fractions of an amount: \( \frac{1}{3} \times \frac{1}{2} \) is the same as \( \frac{1}{3} \) of \( \frac{1}{2} \).

Encourage children to spot the patterns of what is happening in the multiplication, to support them in unpicking the procedure of multiplying fractions by multiplying the numerators and multiplying the denominators.

Mathematical Talk

Could you use folding paper to calculate \( \frac{2}{3} \times \frac{1}{2} \)? How? Use a piece of paper to model this to a friend.

How are the diagrams similar to folding paper? Which do you find more efficient?

What do you notice about the product of the fractions you have multiplied? What is the procedure to multiply fractions?

Does multiplying two numbers always give you a larger product? Explain why.

Varied Fluency

Dexter is calculating \( \frac{1}{3} \times \frac{1}{2} \) by folding paper. He folds a piece of paper in half. He then folds the half into thirds. He shades the fraction of paper he has created. When he opens it up he finds he has shaded \( \frac{1}{6} \) of the whole piece of paper.

\( \frac{1}{3} \times \frac{1}{2} \) means \( \frac{1}{3} \) of a half. Folding half the paper into three equal parts showed me that \( \frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \)

Represent and calculate the multiplications by folding paper.

\[ \frac{1}{4} \times \frac{1}{2} = \quad \frac{1}{4} \times \frac{1}{3} = \quad \frac{1}{4} \times \frac{1}{4} = \]

Alex is drawing diagrams to represent multiplying fractions.

Shade the diagrams to calculate:

\( \frac{1}{3} \times \frac{1}{2} = \quad \frac{1}{4} \times \frac{1}{2} = \quad \frac{1}{3} \times \frac{1}{4} = \quad \frac{2}{3} \times \frac{1}{4} = \quad \frac{2}{3} \times \frac{3}{4} = \)

Write your answers in their simplest form.
Multiply Fractions by Fractions

Reasoning and Problem Solving

The shaded square in the grid below is the answer to a multiplying fractions question. What was the question?

Find the area of the shaded part of the shape.

Find:
\[
\frac{1}{6} \times \frac{1}{4}
\]

Possible answers:
\[
\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}
\]
\[
\frac{2}{2} \times \frac{3}{6} = \frac{6}{12} = \frac{1}{2}
\]
Children could also use improper fractions.

Alex says,
\[
\frac{1}{4} \times \frac{1}{2}
\]
is the same as \(\frac{1}{2}\) of a quarter.

Do you agree? Explain why.

Alex is correct. Multiplication is commutative so
\[
\frac{1}{4} \times \frac{1}{2}
\]
is the same as \(\frac{1}{2}\) of a quarter or \(\frac{1}{4}\) of a half.

The shaded area is \(\frac{11}{21}\) m².
Divide Fractions by Integers (1)

Notes and Guidance

Children are introduced to dividing fractions by integers for the first time. They focus on dividing fractions where the numerator is a multiple of the integer they are dividing by. Encourage children to spot the pattern that the denominator stays the same and the numerator is divided by the integer.

Children link dividing fractions to multiplying by unit fractions. Use the diagrams children drew for multiplying fractions to discuss how and why the calculations are similar.

Mathematical Talk

How could you represent this fraction?
Is the numerator divisible by the integer?
Why doesn't the denominator change?

What pattern can you see when dividing elevenths?
How can we use the pattern to help us to calculate a mixed number by an integer? Can you convert it to an improper fraction?

Varied Fluency

Dexter has \( \frac{2}{5} \) of a chocolate bar. He shares it with his friend. What fraction of the chocolate bar do they each get?

Use the diagrams to help you calculate.

\[
\frac{3}{4} \div 3 = \quad \frac{4}{7} \div 4 = \quad \frac{4}{7} \div 2 =
\]

Calculate.

\[
\frac{1}{11} \div 1 = \quad \frac{2}{11} \div 2 = \quad \frac{3}{11} \div 3 = \quad \frac{4}{11} \div 4 =
\]

\[
\frac{2}{11} \div 2 = \quad \frac{4}{11} \div 2 = \quad \frac{6}{11} \div 2 = \quad \frac{8}{11} \div 2 =
\]

\[
\frac{3}{11} \div 3 = \quad \frac{6}{11} \div 3 = \quad \frac{9}{11} \div 3 = \quad 1 \frac{1}{11} \div 3 =
\]
## Divide Fractions by Integers (1)

### Reasoning and Problem Solving

Tommy says,

Dividing by 2 is the same as finding half of a number so $\frac{4}{11} \div 2$ is the same as $\frac{1}{2} \times \frac{4}{11}$.

Do you agree? Explain why.

<table>
<thead>
<tr>
<th>Match the equivalent calculations.</th>
<th>Tommy is correct. It may help children to understand this by reinforcing that $\frac{1}{2} \times \frac{4}{11}$ is the same as $\frac{1}{2}$ of $\frac{4}{11}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{4} \times \frac{12}{13}$</td>
<td>$\frac{12}{13} \div 2$</td>
</tr>
<tr>
<td>$\frac{1}{6} \times \frac{12}{13}$</td>
<td>$\frac{12}{13} \div 6$</td>
</tr>
<tr>
<td>$\frac{1}{2} \times \frac{12}{13}$</td>
<td>$\frac{12}{13} \div 4$</td>
</tr>
<tr>
<td>$\frac{1}{3} \times \frac{12}{13}$</td>
<td>$\frac{12}{13} \div 3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Complete the missing integers.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{15}{16} \div □ = \frac{5}{16}$</td>
<td>3</td>
</tr>
<tr>
<td>$\frac{15}{16} \div □ = \frac{3}{16}$</td>
<td>5</td>
</tr>
<tr>
<td>$\frac{20}{23} \div □ = \frac{4}{23}$</td>
<td>5</td>
</tr>
<tr>
<td>$\frac{20}{23} \div □ = \frac{5}{23}$</td>
<td>4</td>
</tr>
</tbody>
</table>

Rosie walks for $\frac{3}{4}$ of an hour over 3 days. She walks for the same amount of time each day. How many minutes does Rosie walk each day?

Rosie walks for $\frac{1}{4}$ of an hour each day. She walks for 15 minutes each day.
**Divide Fractions by Integers (2)**

**Notes and Guidance**

Children divide fractions where the numerator is not a multiple of the integer they are dividing by. They draw diagrams to divide fractions into equal parts and explore the link between multiplying by a unit fraction and dividing by an integer. Children find equivalent fractions to support the divisions and draw diagrams to model how this works.

**Mathematical Talk**

How is Mo’s method of dividing fractions similar to multiplying $\frac{1}{3}$ by $\frac{1}{2}$?

Do you prefer Mo’s or Annie’s method? Explain why.

Why does finding an equivalent fraction help us to divide fractions by integers?

What multiplication can I use to calculate $\frac{3}{5} \div 2$? Explain how you know.

**Varied Fluency**

1. Mo is dividing $\frac{1}{3}$ by 2
   - I have divided one third into 2 equal parts. Each part is worth $\frac{1}{6}$.
   - Draw diagrams to calculate:
     - $\frac{1}{3} \div 3 = \frac{2}{3} \div 3 = \frac{1}{5} \div 3 = \frac{2}{5} \div 3 = $

2. Annie is dividing $\frac{2}{3}$ by 4
   - The numerator isn’t a multiple of the integer I am dividing by so I will find an equivalent fraction to help me divide the numerator equally.
   - Find equivalent fractions to calculate:
     - $\frac{3}{5} \div 2 = \frac{1}{3} \div 3 = \frac{2}{3} \div 3 = $
## Divide Fractions by Integers (2)

### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Alex says,</th>
<th>Alex is wrong, we can divide any fraction by an integer if the numerator is a multiple of the divisor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can only divide a fraction by an integer if the numerator is a multiple of the divisor.</td>
<td></td>
</tr>
</tbody>
</table>

Do you agree? Explain why.

---

<table>
<thead>
<tr>
<th>Calculate the missing fractions and integers.</th>
</tr>
</thead>
</table>
| \[
\begin{align*}
\square \div 4 &= \frac{7}{36} \\
\frac{3}{20} \div \square &= \frac{3}{80} \\
\square \div \square &= \frac{2}{5}
\end{align*}
| 7/9 |
| 4 |

Is there more than one possibility?

There are many possibilities in this last question. Children could look for patterns between the fractions and integers.
Four Rules with Fractions

Notes and Guidance
Children combine the four operations when calculating with fractions. This is a good opportunity to recap the order of operations as children calculate equations with and without brackets. Encourage children to draw bar models to represent worded problems in order to understand which operation they need to use.

Mathematical Talk
Which part of the equation do we calculate first when we have more than one operation?

What do you notice about the six questions that begin with $3\frac{1}{3}$?

What's the same about the equations? What's different?

Which equation has the largest answer? Can you order the answers to the equations in descending order?

Can you write the worded problem as a number sentence?

Varied Fluency

Complete the missing boxes.

Calculate:

$3\frac{1}{3} + \frac{1}{3} - 2 = \quad 3\frac{1}{3} + \frac{1}{3} + 2 = \quad 3\frac{1}{3} + \frac{1}{3} \times 2 =$

$3\frac{1}{3} + \frac{1}{3} \div 2 = \quad (3\frac{1}{3} + \frac{1}{3}) \times 2 = \quad (3\frac{1}{3} + \frac{1}{3}) \div 2 =$

Jack has one quarter of a bag of sweets and Whitney has two thirds of a bag of sweets. They combined their sweets and shared them equally between themselves and Rosie. What fraction of the sweets does each child receive?
Four Rules with Fractions

**Reasoning and Problem Solving**

Add two sets of brackets to make the following calculation correct:

\[
\frac{1}{2} + \frac{1}{4} \times 8 + \frac{1}{6} \div 3 = 6\frac{1}{18}
\]

Explain where the brackets go and why. Did you find any difficulties?

Match each calculation to the correct answer.

\[
\left(\frac{2}{3} + \frac{2}{9}\right) \div 4 = \frac{2}{9}
\]

\[
\frac{2}{3} - \frac{1}{3} \div 3 = \frac{5}{9}
\]

\[
\frac{1}{3} \times 2 - \left(1\frac{1}{9} \div 2\right) = \frac{1}{9}
\]

\[
\left(\frac{2}{3} + \frac{2}{9}\right) \div 4 = \frac{2}{9}
\]
Fraction of an Amount

Notes and Guidance

Children calculate fractions of an amount. They recognise that the denominator is the number of parts the amount is being divided into, and the numerator is the amount of those parts we need to know about.

Encourage children to draw bar models to support the procedure of dividing by the denominator and multiplying by the numerator to find fractions of amounts.

Mathematical Talk

What is the value of the whole?
How many equal parts are there altogether?
How many equal parts do we need?
What is the value of each equal part?
Can you see a pattern in the questions starting with $\frac{1}{5}$ of 30?
What would the next column to the right of the questions be?
What would the next row of questions underneath be? How do you know? How can you predict the answers?

Varied Fluency

A cook has 48 kg of potatoes. He uses $\frac{5}{8}$ of the potatoes. How many kilograms of the potatoes does he have left?
Use the bar model to find the answer to this question.

A football team has 300 tickets to give away. They give $\frac{3}{4}$ of them to a local school. How many tickets are left?

Calculate:
$\frac{1}{5}$ of 30 = $\frac{1}{5}$ of 60 = $\frac{1}{5}$ of 120 = $\frac{1}{5}$ of 240 =
$\frac{2}{5}$ of 30 = $\frac{1}{5}$ of 600 = $\frac{1}{10}$ of 120 = $\frac{6}{5}$ of 240 =
$\frac{4}{5}$ of 30 = $\frac{1}{5}$ of 6,000 = $\frac{1}{20}$ of 120 = $\frac{11}{5}$ of 240 =
**Fraction of an Amount**

**Reasoning and Problem Solving**

What is the value of A?
What is the value of B?

A = 648
B = 540

Two fashion designers receive \(\frac{3}{8}\) of 208 metres of material.

One of them says: **We each receive 26 m**

Is she correct? Explain your reasoning.

Calculate the missing digits.

\[
\frac{3}{8} \text{ of } 40 = \frac{?}{10} \text{ of } 150
\]

\[
\frac{1}{5} \text{ of } 315 = \frac{?}{8} \text{ of } 72
\]

1

7

She is incorrect because 26 is only one eighth of 208. She needs to multiply her answer by 3 so that they each get 78 m each.
Find the Whole

Notes and Guidance
Children find the whole amount from the known value of a fraction. Encourage children to continue to use bar models to support them in representing the parts and the whole. Children will consider looking for patterns when calculating the whole. Highlight the importance of multiplication and division when calculating fractions of amounts and how knowing our times-tables can support us to calculate the whole more efficiently.

Mathematical Talk
How many equal parts are there altogether?
How many equal parts do we know?
What is the value of each equal part?
What is the value of the whole?
Can you see a pattern in the questions?
How can we find the whole?
Can you estimate what the answer is? Can you check the answer using a bar model?

Varied Fluency

Jack has spent $\frac{2}{3}$ of his money.
He spent £60, how much did he have to start with?

£60

Use a bar model to represent and solve the problems.

- Rosie eats $\frac{2}{5}$ of a packet of biscuits. She eats 10 biscuits. How many biscuits were in the original packet?
- In an election $\frac{3}{8}$ of a town voted. If 120 people voted, how many people lived in the town?

Calculate:

$\frac{1}{4}$ of ____ = 12  
$\frac{1}{4}$ of ____ = 36  
$\frac{1}{4}$ of ____ = 108

$$\frac{1}{12}$$ of ____ = 12  
$\frac{3}{4}$ of ____ = 36  
$\frac{4}{4}$ of ____ = 108
Find the Whole

Reasoning and Problem Solving

Eva lit a candle while she had a bath. After her bath, 2/5 of the candle was left. It measured 13 cm. Eva says:

Before my bath the candle measured 33 cm

Is she correct? Explain your reasoning.

She is incorrect. 13 ÷ 2 = 6.5
6.5 × 5 = 32.5 cm

She either didn’t halve correctly or didn’t multiply correctly

Write a problem which this bar model could represent.

Many possibilities. 5/8 of children have blue eyes. 15 children do not have blue eyes. How many children are there altogether?

To work out how much we had originally, we should divide 300 by 6 then multiply by 7

No, we know that 300 ml is 1/7 so we need to multiply it by 7

Who is correct? Explain your reasoning.

Rosie and Jack are making juice. They use 6/7 of the water in a jug and are left with this amount of water:

Rosie is correct. Jack would only be correct if 6/7 was remaining but 6/7 is what was used. Rosie recognised that 1/7 is left in the jug therefore multiplied it by 7 to correctly find the whole.