Spring Scheme of Learning

Year 5/6

#MathsEveryoneCan

2019-20
Notes and Guidance

How to use the mixed-age SOL

In this document, you will find suggestions of how you may structure a progression in learning for a mixed-age class.

Firstly, we have created a yearly overview.

Each term has 12 weeks of learning. We are aware that some terms are longer and shorter than others, so teachers may adapt the overview to fit their term dates.

The overview shows how the content has been matched up over the year to support teachers in teaching similar concepts to both year groups. Where this is not possible, it is clearly indicated on the overview with 2 separate blocks.

For each block of learning, we have grouped the small steps into themes that have similar content. Within these themes, we list the corresponding small steps from one or both year groups. Teachers can then use the single-age schemes to access the guidance on each small step listed within each theme.

The themes are organised into common content (above the line) and year specific content (below the line). Moving from left to right, the arrows on the line suggest the order to teach the themes.
How to use the mixed-age SOL

Here is an example of one of the themes from the Year 1/2 mixed-age guidance.

<table>
<thead>
<tr>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1 (Aut B2, Spr B1)</td>
</tr>
<tr>
<td>- How many left? (1)</td>
</tr>
<tr>
<td>- How many left? (2)</td>
</tr>
<tr>
<td>- Counting back</td>
</tr>
<tr>
<td>- Subtraction - not crossing 10</td>
</tr>
<tr>
<td>- Subtraction - crossing 10 (1)</td>
</tr>
<tr>
<td>- Subtraction - crossing 10 (2)</td>
</tr>
<tr>
<td>Year 2 (Aut B2, B3)</td>
</tr>
<tr>
<td>- Subtract 1-digit from 2-digits</td>
</tr>
<tr>
<td>- Subtract with 2-digits (1)</td>
</tr>
<tr>
<td>- Subtract with 2-digits (2)</td>
</tr>
<tr>
<td>- Find change - money</td>
</tr>
</tbody>
</table>

In order to create a more coherent journey for mixed-age classes, we have re-ordered some of the single-age steps and combined some blocks of learning e.g. Money is covered within Addition and Subtraction.

The bullet points are the names of the small steps from the single-age SOL. We have referenced where the steps are from at the top of each theme e.g. Aut B2 means Autumn term, Block 2. Teachers will need to access both of the single-age SOLs from our website together with this mixed-age guidance in order to plan their learning.

Points to consider

- Use the mixed-age schemes to see where similar skills from both year groups can be taught together. Learning can then be differentiated through the questions on the single-age small steps so both year groups are focusing on their year group content.
- When there is year group specific content, consider teaching in split inputs to classes. This will depend on support in class and may need to be done through focus groups.
- On each of the block overview pages, we have described the key learning in each block and have given suggestions as to how the themes could be approached for each year group.
- We are fully aware that every class is different and the logistics of mixed-age classes can be tricky. We hope that our mixed-age SOL can help teachers to start to draw learning together.
## WRM – Year 5/6 – Scheme of Learning 2.0s

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
<th>Week 8</th>
<th>Week 9</th>
<th>Week 10</th>
<th>Week 11</th>
<th>Week 12</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Autumn</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Number: Place Value</td>
<td>Number: Four Operations</td>
<td>Number: Fractions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>Spring</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y5: Number: Fractions</td>
<td>Number: Decimals and Percentages</td>
<td>Y5: Number: Decimals</td>
<td>Measurement: Converting Units</td>
<td>Measurement: Perimeter, Area and Volume</td>
<td>Statistics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y6: Number: Ratio</td>
<td>Y6: Number: Algebra</td>
<td></td>
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<td></td>
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<td></td>
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<tr>
<td><strong>Summer</strong></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y6: SATS</td>
<td>Investigations</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In this section, content from single-age blocks are matched together to show teachers where there are clear links across the year groups.

Teachers may decide to teach the lower year’s content to the whole class before moving the higher year on to their age-related expectations.

The lower year group is not expected to cover the higher year group’s content as they should focus on their own age-related expectations.

In this section, content that is discrete to one year group is outlined.

Teachers may need to consider a split input with lessons or working with children in focus groups to ensure they have full coverage of their year’s curriculum.

Guidance is given on each page to support the planning of each block.

The themes should be taught in order from left to right.
Decimals and Algebra

Common Content

Year 5 and 6 are studying different topics in this unit.

Teachers may decide to recap adding and subtracting decimals with Year 6. This can then be applied throughout other topics including in their algebra block.

Year Specific

Decimals
Year 5 (Sum B1)
• Adding decimals within 1
• Subtracting decimals within 1
• Complements to 1
• Adding decimals - crossing the whole
• Adding decimals (same d.p.)
• Subtracting decimals (same d.p.)
• Adding decimals (different d.p.)
• Subtracting decimals (different d.p.)
• Adding and subtracting wholes and decimals
• Decimal sequences

Algebra
Year 6 (Spr B3)
• Find a rule- one step
• Find a rule- two steps
• Forming expressions
• Substitution
• Formulae
• Forming equations
• Simple one-step equations
• Solve two-step equations
• Find pairs of values
• Enumerate possibilities
Block 3 - Decimals and Algebra

Year 5 - Decimals
Children add decimals within one whole. They use place value counters and place value charts to support adding decimals and understand what happens when we exchange between columns.

Children build on their understanding that 0.45 is 45 hundredths, children can use a hundred square to add decimals.

What is the number represented on the place value chart? What digit changes when I add a hundredth?

Each box in this hundred square represents one hundredth of the whole. Use this to answer:

- What number is one hundredth more?
- Add 0.3, what number do you have now?
- How many more thousandths can I add before the hundredths digit changes?

Use this place value chart to help answer the questions.

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.1</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Use the column method to complete the additions.

- 0.07 + 0.78
- 0.87 + 0.07
- 0.45 + 0.5
- 0.45 + 0.05
- 0.45 + 0.005
What mistake has Dora made?

Dora has put the 3 tenths in the thousandths place. The correct answer is 0.71.

Use at least 2 representations to show why she is incorrect.

Compare the numbers sentences using <, > or =

<table>
<thead>
<tr>
<th>0.7 + 0.03 + 0.001</th>
<th>0.07 + 0.3 + 0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;</td>
<td></td>
</tr>
</tbody>
</table>

Rosie has some digit cards. She uses each card once to make a number sentence.

Largest: 0.951
Smallest: 0.159

What is the largest number she can make? What is the smallest?

0.41 + 0.3 = 0.413
0.7 + 0.03 + 0.001 = 0.731
0.4 + 0.1 + 0.05 = 0.55
0.3 + 0.2 + 0.05 = 0.57
Children subtract decimals using a variety of different methods. They look at subtracting using place value counters on a place value grid. Children also explore subtraction as difference by using a number line to count on from the smaller decimal to the larger decimal.

Children use their knowledge of exchange within whole numbers to subtract decimals efficiently.

### Notes and Guidance

**Mathematical Talk**

- What is the number represented on the place value chart?
- What is one tenth less than one?
- What is one hundredth less than one?
- Show me how you know.
- If I’m taking away tenths, which digit will be affected? Is this always the case?
- How many hundredths can I take away before the tenths place is affected?

### Varied Fluency

Here is a number.

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
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<tr>
<td></td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
</tr>
</tbody>
</table>

- What is three tenths less than the number?
- Take away 0.02, what is your number now?
- Subtract 5 thousandths. What is the final number?

Find the difference between the two numbers using the number line.

\[0.424 - 0.618\]

#### Calculate.

\[0.584 - 0.154 = 0.44 - 0.1 = \]
\[0.684 - 0.254 = 0.44 - 0.09 = \]
\[0.685 - 0.255 = 0.44 - 0.11 = \]
Here are four calculations. Which one is the easiest to answer? Which one is the trickiest to answer? Explain your choice of order.

0.45 − 0.3 =
0.45 − 0.15 =
0.45 − 0.23 =
0.45 − 0.18 =

Children justify the order they have given.

Possible order:
0.45 − 0.23 = 0.22
(no exchange)
0.45 − 0.15 = 0.3
(no exchange with 0)
0.45 − 0.3 = 0.15
(no exchange, different dp)
0.45 − 0.18 = 0.27
(exchange)

The strip of paper is 0.8 m long.

It is cut into two unequal parts.

The difference in lengths between the two strips of paper is 0.1 m

How long are the two strips of paper?

Strip 1: 0.45 m
Strip 2: 0.35 m
Complements to 1

Notes and Guidance

Children find the complements which sum to make 1

It is important for children to see the links with number bonds to 10, 100 and 1000

This will support them when finding complements to 1, up to three decimal places.

Children can use a hundred square, part-whole models and number lines to support finding complements to one.

Mathematical Talk

What number bonds can you use to help you?

How can shading the hundred square help you find the complement to 1?

How many different ways can you make 1? How many ways do you think there are?

If I add ______, which place will change? How many can I add to change the tenths/hundredths place?

Varied Fluency

Using a blank hundred square, where each square represents one hundredth, find the complements to 1 for these numbers.

0.55 + □ = 1

1 = 0.32 + □

0.11 + 0.5 + □ = 1

Complete the part-whole models.

Use the number line to find the complements to 1

0.324 _____________________________ 1

0.459 _____________________________ 1
Do you agree with Tommy? Can you explain what his mistake was?

Tommy has forgotten that when you have ten in a place value column you need to use your rules of exchanging.

e.g.
10 tenths = 1 one
10 hundredths = 1 tenth
10 thousandths = 1 hundredth

The correct answer is 0.667

How many different ways can you find a path through the maze, adding each number at a time, to make a total of one?

Once you have found a way, can you design your own smaller maze for others to solve?
Adding – Crossing the Whole

**Notes and Guidance**

Children use their skills at finding complements to 1 to support their thinking when crossing the whole. Children require flexibility at partitioning decimals, as bridging will be extremely important. Encourage children to make one first, then add the remaining decimal.

For example: 0.74 + 0.48 =

\[
0.74 + 0.26 + 0.22 = 1.22
\]

**Mathematical Talk**

What happens when we have 10 in a place value column?

How would partitioning a number help us?

How do you decide what number to partition?

Why is partitioning 0.67 into 0.55 and 0.12 more helpful than 0.6 and 0.07?

What complement to 1 would I use to answer this question?

**Varied Fluency**

- Use the place value grid to answer 0.453 + 0.664

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.001</td>
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<td>0.01</td>
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<tr>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.001</td>
</tr>
</tbody>
</table>

- Amir is using complements to 1 to add decimals.

\[
0.45 + 0.67 = \\
0.55 + 0.12
\]

\[
0.45 + 0.55 + 0.12 = 1.12
\]

- Use Amir’s method to solve:
  a) 0.56 + 0.78
  b) 3.42 + 0.79

- Use the column method to solve the additions:

\[
0.47 + 0.6 \\
0.982 + 0.18 \\
0.92 + 0.8
\]
Adding – Crossing the Whole

Reasoning and Problem Solving

A place value grid is used to solve $0.7 + 0.5$

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.1</td>
</tr>
<tr>
<td>.1</td>
<td>.2</td>
</tr>
<tr>
<td>.3</td>
<td>.3</td>
</tr>
<tr>
<td>.3</td>
<td>.4</td>
</tr>
<tr>
<td>.5</td>
<td>.6</td>
</tr>
<tr>
<td>.7</td>
<td>.8</td>
</tr>
</tbody>
</table>

Alex thinks the answer is 0.12

What mistake has she made?

Ten lots of one tenth is one whole. There are 12 tenths so Alex needs to make an exchange. She should exchange 10 tenths for 1 one.

The correct answer is 1.2

Ten lots of one tenth is one whole. There are 12 tenths so Alex needs to make an exchange. She should exchange 10 tenths for 1 one.

The correct answer is 1.2

You will need a partner and a six-sided dice for this game.

Take it in turns rolling the dice twice and placing the digits in the blank spaces above. Record the number in a table.

Swap over with your partner.

Roll the dice again and add your new number to the first number. The winner is the person who after adding 4 numbers is the closest to 1.5 without going over.

Example:

Player 1 rolls a 1 and a 4. 0.14

Player 1 then rolls a 2 and a 6. 0.26

0.14 + 0.26 = 0.38

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.14</td>
<td>0.64</td>
</tr>
<tr>
<td>0.38</td>
<td>1.23</td>
</tr>
<tr>
<td>0.69</td>
<td>1.49</td>
</tr>
<tr>
<td>1.24</td>
<td>1.60</td>
</tr>
</tbody>
</table>
Children add numbers greater than one with the same number of decimal places.

Place value grids and counters are extremely helpful in ensuring children are understanding the value of each digit and understanding when to exchange.

Ensure children see the formal written method (column addition) alongside the place value chart.

Mathematical Talk

Why is it important to line up the columns?

What happens when there are a total of ten counters in a place value column?

Why is the position of the decimal point important?

Use the place value chart to add 3.45 and 4.14

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1</td>
<td>0.1 1</td>
<td>0.01 0.01</td>
</tr>
<tr>
<td>1 1</td>
<td>0.1 1</td>
<td>0.01 0.01</td>
</tr>
</tbody>
</table>

3 . 4 5
+ 4 . 1 4
________________

Use the column method to solve these additions.

4 . 4 2
+ 7 . 6 3
________________

4 . 5 5
+ 3 . 0 7
________________

Ron goes to the shops. He buys 3 items. What is the most he could pay? What is the least he could pay?

£4.45 £5.59 £3.99 £4.05
Adding – Same Decimal Places

Reasoning and Problem Solving

Using these strategies, can you find more number sentences which have the same total as $3 + 3$

Children may find a range of answers. The important teaching point is to highlight that you have added the same to one number as you have taken away from the other.

Using the digits 0 – 9 only once in each of the spaces above, what is:
• The largest sum possible
• The smallest sum possible

Is there more than one way of creating each total?

- Largest
  - 9.75 + 8.64
  - 9.65 + 8.74
  - 9.64 + 8.75
  - 9.74 + 8.65

- Smallest
  - 0.24 + 1.35
  - 0.25 + 1.34
  - 0.34 + 1.25
  - 0.35 + 1.24
Notes and Guidance

Children subtract numbers with the same number of decimal places. They use place value counters and a place value grid to support them with exchanging.

Children should be given opportunities to apply subtraction to real life contexts which could involve measures. Bar models can be a useful representation of the problems.

Mathematical Talk

What happens when you need to subtract a greater digit from a smaller digit e.g. 3 hundredths subtract 4 hundredths?

How many tenths are equivalent to one hundredth?

Do we only ever make one exchange in a subtraction calculation?

Which of these numbers will need exchanging?

Can you predict what the answer might be?

How could you check your answer?

Varied Fluency

Use the place value chart to find the answer 4.33 - 2.14

```
  4 . 3 3
- 2 . 1 4
  ___
```

Use the column method to answer these questions.

```
  6 . 4
- 3 . 8
  ___
```

```
  5 . 0 5
- 2 . 1 5
  ___
```

Jack has £12.54 in his wallet. He buys a football which costs £5.82

How much money does he have left?
Dexter and Annie have some money. Dexter has £3.45 more than Annie.

They have £12.45 altogether.

How much money does Annie have?

Annie has £4.50

In this number pyramid, each number is calculated by adding the two numbers underneath.
Children add numbers with different numbers of decimal places. They focus on the importance of lining up the decimal point in order to ensure correct place value.

Children should be encouraged to think about whether their answers are sensible. For example, when adding 1.3 to 1.32 and getting an answer 1.45, how do we know it is not a sensible answer? Discuss the importance of estimation.

Why is the decimal point important when we are reading and writing a number?

What would a sensible estimate be?

Is this a sensible answer? Why/why not?

What advice would you give to someone that is struggling with recording their numbers in the correct place?

Use the place value grid to add 1.3 and 3.52

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\[
1.3 + 3.52 = 4.82
\]

Use the column method to answer these questions.

\[
4.4 + 7.044 + 1.6 = 12.104
\]

Whitney is cycling in a race. She has cycled 3.145 km so far and has 4.1 km left to go. What is the total distance of the race?
Eva is trying to find the answer to

\[ 4.144 + 1.4 \]

Here is her working out.

\[
\begin{align*}
4 &. 1 4 4 \\
+ &. 1 4 \\
\hline
4 &. 2 4 8
\end{align*}
\]

Can you spot and explain her error?

The digits are lined up incorrectly.

Eva needs to line up the decimal point.

The correct answer is 5.544

Work out the correct answer.

Place the calculations in the correct column in the table.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>No exchange</th>
<th>Exchange in the ones column</th>
<th>Exchange in the tenths column</th>
<th>Exchange in the hundredths column</th>
<th>Exchange in the thousandths column</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ 9.99 + 0.1 ]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 9.99 + 1 ]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 9.99 + 0.001 ]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ 9.99 + 0.01 ]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Some calculations might need to go in more than one place.

Add 2 more calculations to each column.

No exchange: 9.99 + 0.001
Exchange in the ones column: 9.99 + 1
9.99 + 0.1
9.99 + 0.01
Exchange in the tenths column: 9.99 + 0.1
9.99 + 0.01
Exchange in the hundredths column: 9.99 + 0.01
Subtracting – Different D.P.

Notes and Guidance

Children subtract decimals with different numbers of decimal places.

They continue to focus on the importance of lining up the decimal point in order to ensure correct place value.

Children identify the importance of zero as a place holder.

Mathematical Talk

What does it mean if there is nothing in a place value column? How can we represent this in the formal written method?

What do you notice about $4.7 - 3.825$ and $4.699 - 3.824$? Is one of them more difficult than the other? Why?

Are there more efficient methods for this question?

Varied Fluency

Use the place value grid to help subtract $1.4$ from $4.54$.

<table>
<thead>
<tr>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Use the column method to work out the following.

$6.06 - 3.7 = 3.3 - 1.34 = 14.41 - 1.43 = 3 - 1.87 =$

How much change would I get from £10 if I bought a bag of apples costing £4.27?
Do you agree with Whitney?
Explain your answer.

Whitney is not correct. She needs to use zero as a place value holder in the hundredths column of 4.9 and then exchange.

Encourage children to explore more efficient mental strategies as well as correcting the formal method.

The correct answer is 1.05

Teddy used a calculator to solve:
31.4 \( - \) 1.408

When he looked at his answer of 17.32 he realised he’d made a mistake.

He had typed all the correct digits in.

Can you spot his mistake?
What should the correct answer be?

Teddy placed the decimal point after the 4 making 14.08 instead of 1.408

The correct answer is 29.992
Children add and subtract numbers with decimals from whole numbers. Highlight that whole numbers are written without a decimal point.

There may be a misconception when recording integers, link this to the place value grid. Emphasise prior understanding that the decimal point is to the right of the ones place.

**Mathematical Talk**

What is a whole number/integer?

Where can we add a decimal point to the number 143 so that its value stays the same?

What’s the same and what’s different about 10 and 10.0?

Can you use different methods? (Number line, column subtraction, mentally).

Which is most efficient for this calculation? Explain why.

**Varied Fluency**

Use the place value grid to help add 143 and 1.45

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 143 + 1.45 = \]

Use the place value grid to help work out 12 \(-\) 1.2

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

\[ 12 - 1.2 = \]

Find the most efficient method to solve this calculation.

\[ 43 - 2.14 + 0.86 = \]
\[ 19 - 0.25 = \]
\[ 23 + 4.105 = \]
\[ 19 - 17.37 = \]
What are the missing digits in the calculation?

\[
\begin{array}{c}
31.00 \\
- 1.37 \\
\hline
29.63
\end{array}
\]

Two envelopes contain two different numbers.

- The sum of the numbers is 9.92
- The difference between the numbers is 2.32

What numbers are inside the envelopes?

How can this bar model help?

\[
\begin{array}{c}
a \\
b \\
\hline
9.92
\end{array}
\]

3.8 and 6.12

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Decimal Sequences

Notes and Guidance

Children look at decimal sequences and create simple rules, for example: adding 0.5 every time.

It is important to note that they are not expected to generate algebraic expressions for the sequences, but the use of the word ‘term’ could be used to predict the next number in the sequence. For example, what would be the value of the 10th term in the sequence?

Mathematical Talk

What do increasing and decreasing mean?

Is the sequence increasing by the same amount each time? By how much?

What is the same about each term? What is changing in each term?

What will the next term in the sequence be?

Complete the sequence.

Write the rules for each sequence.

- 0.45, 0.6, 0.75, 0.9 The rule is
- 1.25, 2.5, 3.75, 5, 6.25 The rule is

Generate the first 5 terms of this sequence.

The 1st term is 1.74
The sequence decreases by 0.24 each time.
Do you agree with Jack? Explain your answer.

Jack is incorrect, 9.68 and 9.72 will be in the sequence but not 9.7
The terms are increasing by 0.04 therefore 9.7 will not be in the sequence.

The number 9.7 will be in this sequence.

Eva compared the two sequences above. What do you notice about the differences between the terms in the two sequences?
Investigate Eva’s sequences below and explain your thinking.

<table>
<thead>
<tr>
<th>1st sequence</th>
<th>Relationship</th>
<th>2nd sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st term</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>2nd term</td>
<td>0.2</td>
<td>2</td>
</tr>
<tr>
<td>3rd term</td>
<td>0.3</td>
<td>3</td>
</tr>
<tr>
<td>4th term</td>
<td>0.4</td>
<td>4</td>
</tr>
</tbody>
</table>

The difference between the terms is increasing by 0.9 each time e.g.
1<sup>st</sup> + 0.9
2<sup>nd</sup> + 1.8
3<sup>rd</sup> + 2.7
4<sup>th</sup> + 3.6
Children may also notice that the terms in the 2<sup>nd</sup> sequence are ten times larger than in the first.
The differences would increase by 0.99 each time.

I wonder what the differences would be between sequences that go up in + 0.01 and +1 sequence...
Find a Rule – One Step

Notes and Guidance

Children explore simple one-step function machines. Explain that a one-step function is where they perform just one operation on the input.

Children understand that for each number they put into a function machine, there is an output. They should also be taught to “work backwards” to find the input given the output. Given a set of inputs and outputs, they should be able to work out the function.

Mathematical Talk

What do you think “one-step function” means?
What examples of functions do you know?
Do some functions have more than one name?
What do you think input and output mean?
What is the output if ....?
What is the input if ....?
How many sets of inputs and outputs do you need to be able to work out the function? Explain how you know.

Varied Fluency

Here is a function machine.

Input \[\times 4\] Output

- What is the output if the input is 2?
- What is the output if the input is 7.2?
- What is the input if the output was 20?
- What is the input if the output was 22?

Complete the table for the function machine.

<table>
<thead>
<tr>
<th>Input</th>
<th>5</th>
<th>5.8</th>
<th>10</th>
<th>−3</th>
<th>−8</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>169</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Find the missing function.

- 10 → 5
- 24 → 12
- 7 → 3.5
## Find a Rule – One Step

### Reasoning and Problem Solving

Eva has a one-step function machine. She puts in the number 6 and the number 18 comes out.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

What could the function be? How many different answers can you find?

The function could be $+12$, $\times3$.

Amir puts some numbers into a function machine.

<table>
<thead>
<tr>
<th>Number</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

What is the output from the function when the input is 16?

The function is $\div2$ so the output is $-6$.

Dora puts a number into the function machine.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\div2$</td>
</tr>
</tbody>
</table>

Dora's number is:
- A factor of 32
- A multiple of 8
- A square number

What is Dora's input? What is her output?

Dora's input is 16
Her output is 8

Can you create your own clues for the numbers you put into a function machine for a partner to solve?
Find a Rule – Two Step

Notes and Guidance

Children build on their knowledge of one-step functions to look at two-step function machines. Discuss with children whether a function such as $+5$ and $+6$ is a two-step function machine or whether it can be written as a one-step function. Children look at strategies to find the functions. They can use trial and improvement or consider the pattern of differences. Children record their input and output values in the form of a table.

Mathematical Talk

How can you write $+5$ followed by $-2$ as a one-step function?

If I change the order of the functions, is the output the same?

What is the output if …?

What is the input if …?

If you add 3 to a number and then add 5 to the result, how much have you added on altogether?

Varied Fluency

Here is a function machine.

- What is the output if the input is 5?
- What is the input if the output is 19?
- What is the output if the input is 3.5?

Complete the table for the given function machine.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

- What patterns do you notice in the outputs?
- What is the input if 20 is the output? How did you work it out?

How can you write this two-step machine as a one-step machine?

Check your answer by inputting values.
Teddy has two function machines.

Input → + 5 → × 2 → Output

Input → × 2 → + 5 → Output

He says,

The function machines will give the same answer.

Is Teddy correct?

Is there an input that will give the same output for both machines?

No they do not give the same answer. Encourage children to refer to the order of operations to help them understand why the outputs are different.

Mo has the following function machines.

Input → + 2 → − 8 → Output

Input → × 8 → ÷ 2 → Output

Input → × 2 → − 8 → Output

The first one can be written as − 6
The second can be written as × 4
The third cannot be written as a single machine.

Explain which of these can be written as single function machines.
**Forming Expressions**

**Notes and Guidance**

Children have now met one-step and two-step function machines with numerical inputs. In this step, children use simple algebraic inputs e.g. $y$. Using these inputs in a function machine leads them to forming expressions e.g. $y + 4$. The use of cubes to represent a variable can aid understanding.

Children are introduced to conventions that we use when writing algebraic expressions. e.g. $y \times 4$ as $4y$.

**Mathematical Talk**

What expressions can be formed from this function machine?

What would the function machine look like for this rule/expression?

How can you write $x \times 3 + 6$ differently?

Are $2a + 6$ and $6 + 2a$ the same? Explain your answer.

**Varied Fluency**

Mo uses cubes to write expressions for function machines.

$\text{Input} \rightarrow + 4 \rightarrow \text{Output}$

$\text{Input} \rightarrow \times 4 \rightarrow \text{Output}$

$y \rightarrow y + 4$

$y \rightarrow 4y$

Use Mo's method to represent the function machines. What is the output for each machine when the input is $a$?

$\text{Input} \rightarrow + 2 \rightarrow \text{Output}$

$\text{Input} \rightarrow \times 3 \rightarrow \text{Output}$

$\text{Input} \rightarrow \times 2 \rightarrow + 3 \rightarrow \text{Output}$

$\text{Input} \rightarrow \times 5 \rightarrow + 2 \rightarrow \text{Output}$

Eva is writing expressions for two-step function machines.

Use Eva's method to write expressions for the function machines.
Amir inputs $m$ into these function machines.

He says the outputs of the machines will be the same.

Do you agree?

Explain your answer.

No, because $2m + 1$ isn’t the same as $2m + 2$.

This function machine gives the same output for every input. For example if the input is 5 then the output is 5 and so on.

What is the missing part of the function?

What other pairs of functions can you think that will do the same?

Other pairs of functions that will do the same are functions that are the inverse of each other e.g. $+3, -3$.
Substitution

Notes and Guidance
Children substitute into simple expressions to find a particular value.

They have already experienced inputting into a function machine, and teachers can make the links between these two concepts.

Children will need to understand that the same expression can have different values depending on what has been substituted.

Mathematical Talk
Which letter represents the star?
Which letter represents the heart?
Would it still be correct if it was written as \(a + b + c\)?

What does it mean when a number is next to a letter?

Is \(a + b + b\) the same as \(a + 2b\)?

Varied Fluency
If \(\star = 7\) and \(\heartsuit = 5\), what is the value of:

\[\star + \heartsuit + \heartsuit\]

If \(a = 7\) and \(b = 5\) what is the value of:

\[a + b + b\]

What is the same and what is different about this question?

Substitute the following to work out the values of the expressions.

\[w = 3 \quad x = 5 \quad y = 2.5\]

- \(w + 10\)
- \(w + x\)
- \(y - w\)

Substitute the following to work out the values of the expressions.

\[w = 10 \quad x = \frac{1}{4} \quad y = 2.5\]

- \(3y\)
- \(wx\)
- \(12 + 8.8w\)
- \(wy + 4x\)
Here are two formulae.

\[ p = 2a + 5 \]
\[ c = 10 - p \]

Find the value of \( c \) when \( a = 10 \)

\[ c = 10 - (2 \times 10 + 5) \]
\[ c = 10 - 25 \]
\[ c = -15 \]

\[ x = 2c + 6 \]

Whitney says, No Whitney is incorrect. \( c \) could have any value.

Amir says, When \( c = 5 \), \( x = 31 \)

Amir is wrong. Explain why. What would the correct value of \( x \) be?

\[ x = 2 \times 5 + 6 \]
\[ x = 12 \]

Amir has put the 2 next to the 5 to make 25 instead of multiplying 2 by 5. The correct value of \( x \) would be 16.
Which of the following is a formula?

- \( P = 2l + 2w \)
- \( 3d + 5 \)
- \( 20 = 3x - 2 \)

Explain why the other two are not formulae.

Eva uses the formula \( P = 2l + 2w \) to find the perimeter of rectangles.
Use this formula to find the perimeter of rectangles with the following lengths and widths.
- \( l = 15, w = 4 \)
- \( l = \frac{1}{4}, w = \frac{3}{8} \)
- \( l = w = 5.1 \)

This is the formula to work out the cost of a taxi.

\[ C = 1.50 + 0.3m \]

- \( C \) = the cost of the journey in £
- \( m \) = number of miles travelled.

Work out the cost of a 12-mile taxi journey.
Jack and Dora are using the following formula to work out what they should charge for four hours of cleaning.

Cost in pounds = 20 + 10 \times \text{number of hours}

Jack thinks they should charge £60
Dora thinks they should charge £120

Who do you agree with? Why?

Jack is correct as multiplication should be performed first following the order of operations.

Dora has not used the order of operations – she has added 20 and 10 and then multiplied 30 by 4

The rule for making scones is use 4 times as much flour ($f$) as butter ($b$).

Which is the correct formula to represent this?

A. $f = \frac{b}{4}$
B. $f = 4b$
C. $f = b + 4$
D. $4f = b$

Explain why the others are incorrect.

B is correct.
A shows the amount of flour is a quarter of the amount of butter.
C shows the amount of flour is 4 more than butter.
D shows butter is 4 times the amount of flour.
Forming Equations

Notes and Guidance

Building on the earlier step of forming expressions, children now use algebraic notation to form one-step equations. They need to know the difference between an expression like \( x + 5 \), which can take different values depending on the value of \( x \), and an equation like \( x + 5 = 11.2 \) where \( x \) is a specific unknown value. This is best introduced using concrete materials e.g. cubes, can be used to represent the unknown values with counters being used to represent known numbers.

Mathematical Talk

What does the cube represent?
What do the counters represent?

Design your own ‘think of a number’ problems.

What’s the difference between an expression and an equation?
What’s the difference between a formula and an equation?

Varied Fluency

Amir represents a word problem using cubes, counters and algebra.

<table>
<thead>
<tr>
<th>Words</th>
<th>Concrete</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>I think of a number</td>
<td>![Cube]</td>
<td>( x )</td>
</tr>
<tr>
<td>Add 3</td>
<td>![Counters]</td>
<td>( x + 3 )</td>
</tr>
<tr>
<td>My answer is 5</td>
<td>![Equation]</td>
<td>( x + 3 = 5 )</td>
</tr>
</tbody>
</table>

Complete this table using Amir’s method.

<table>
<thead>
<tr>
<th>Words</th>
<th>Concrete</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>I think of a number</td>
<td>![Cube]</td>
<td></td>
</tr>
<tr>
<td>Add 1</td>
<td>![Counters]</td>
<td></td>
</tr>
<tr>
<td>My answer is 8</td>
<td>![Counters]</td>
<td></td>
</tr>
</tbody>
</table>

A book costs £5 and a magazine costs \( \$n \)
The total cost of the book and magazine is £8
Write this information as an equation.

Write down algebraic equations for these word problems.
- I think of a number, subtract 17, my answer is 20
- I think of a number, multiply it by 5, my answer is 45
### Forming Equations

#### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Rosie thinks of a number. She adds 7 and divides her answer by 2</th>
<th>They both think of 11, therefore Teddy’s answer is 29</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teddy thinks of a number. He multiples by 3 and subtracts 4</td>
<td>Rosie and Teddy think of the same number. Rosie’s answer is 9. What is Teddy’s answer?</td>
</tr>
<tr>
<td>Rosie and Teddy think of the same number again. This time, they both get the same answer.</td>
<td>They think of 3 and the answer they both get is 5</td>
</tr>
<tr>
<td>Use trial and improvement to find the number they were thinking of.</td>
<td>Eva spends 92p on yo-yos and sweets. She buys $y$ yo-yos costing 11p and $s$ sweets costing 4p. Can you write an equation to represent what Eva has bought? How many yo-yos and sweets could Eva have bought?</td>
</tr>
<tr>
<td>92 = 11y + 4s</td>
<td>Can you write a similar word problem to describe this equation? 74 = 15t + 2m</td>
</tr>
</tbody>
</table>
One-step Equations

Notes and Guidance

Children solve simple one-step equations involving the four operations.

Children should explore this through the use of concrete materials such as cubes, counters and cups.

It is recommended that children learn to solve equations using a balancing method using inverse operations.

Mathematical Talk

Can you make some of your own equations using cups and counters for a friend to solve?

Why do you think the equation is set up on a balance? What does the balance represent? How does this help you solve the equation?

What is the same and what is different about each bar model?

Varied Fluency

How many counters is each cup worth?
Write down and solve the equation represented by the diagram.

Solve the equation represented on the scales.
Can you draw a diagram to go with the next step?

Match each equation to the correct bar model and then solve to find the value of x.

\[ x + 5 = 12 \]
\[ 3x = 12 \]
\[ 12 = 3 + x \]
One-step Equations

Reasoning and Problem Solving

The perimeter of the triangle is 216 cm.

Form an equation to show this information.

Solve the equation to find the value of \( x \).

Work out the lengths of the sides of the triangle.

\[ 3x + 4x + 5x = 216 \]
\[ 12x = 216 \]
\[ x = 18 \]

\[ 5 \times 18 = 90 \]
\[ 3 \times 18 = 54 \]
\[ 4 \times 18 = 72 \]

- Hannah is 8 years old
- Jack is 13 years old
- Grandma is \( x + 12 \) years old.
- The sum of their ages is 100

Form and solve an equation to work out how old Grandma is.

\[ 8 + 13 + x + 12 = 100 \]
\[ 33 + x = 100 \]
\[ x = 77 \]

Grandma is 77 years old.

What is the size of the smallest angle in this isosceles triangle?

8 \( y \) = 180
\[ y = 22.5 \]

Smallest angle = 45°
Check by working them all out and see if they add to 180°
Two-step Equations

Notes and Guidance

Children progress from solving equations that require one-step to equations that require two steps. Children should think of each equation as a balance and solve it through doing the same thing to each side of the equation. This should be introduced using concrete and pictorial methods alongside the abstract notation as shown. Only when secure in their understanding should children try this without the support of bar models or similar representations.

Mathematical Talk

Why do you have to do the same to each side of the equation?

Why subtract 1? What does this do to the left hand side of the equation?

Does the order the equation is written in matter?

What’s the same and what’s different about solving the equations $2x + 1 = 17$ and $2x - 1 = 17$?

Varied Fluency

Here is each step of an equation represented with concrete resources.

$\begin{align*}
2x + 1 &= 5 \\
-1 &-1 \\
2x &= 4 \\
+2 &+2 \\
x &= 2
\end{align*}$

Use this method to solve:

$4y + 2 = 6$  
$9 = 2x + 5$  
$1 + 5a = 16$

Here is each step of an equation represented by a bar model. Write the algebraic steps that show the solution of the equation.

Use bar models to solve these equations.

$3b + 4 = 19$  
$20 = 4b + 2$
The length of a rectangle is $2x + 3$
The width of the same rectangle is $x - 2$
The perimeter is 17 cm.

Find the area of the rectangle.

Alex has some algebra expression cards.

$y + 4$
$2y$
$3y - 1$

The mean of the cards is 19
Work out the value of each card.

Here is the quadrilateral ABCD.

The perimeter of the quadrilateral is 80 cm.

$4y + 1 = 21$
$4y = 20$
y = 5

AB = 21 cm
BC = 21 cm
AD = 26 cm
CD = 80 - (21 + 21 + 26) = 12 cm
Find Pairs of Values (1)

Notes and Guidance

Children use their understanding of substitution to consider what possible values a pair of variables can take.

At this stage we should focus on integer values, but other solutions could be a point for discussion.

Children can find values by trial and improvement, but should be encouraged to work systematically.

Mathematical Talk

Can \(a\) and \(b\) be the same value?

Is it possible for \(a\) or \(b\) to be zero?

How many possible integer answers are there? Convince me you have them all.

What do you notice about the values of \(c\) and \(d\)?

Varied Fluency

\[a + b = 6\]

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

There are lots of possible solutions to this equation. Find 5 different possible integer values for \(a\) and \(b\).

\(X\) and \(Y\) are whole numbers.
- \(X\) is a one digit odd number.
- \(Y\) is a two digit even number.
- \(X + Y = 25\)

Find all the possible pairs of numbers that satisfy the equation.

\[c \times d = 48\]

<table>
<thead>
<tr>
<th>(c)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What are the possible integer values of \(c\) and \(d\)?

How many different pairs of values can you find?
Find Pairs of Values (1)

Reasoning and Problem Solving

\(a, b\) and \(c\) are integers between 0 and 5

\[a + b = 6\]
\[b + c = 4\]

Find the values of \(a, b\) and \(c\)

How many different possibilities can you find?

Possible answers:

\[
\begin{align*}
\text{Possible answers:} & \\
\text{Possible answer:} & \\
\text{Dora is correct as} & \\
\text{Jack says,} & \\
\text{Only one is correct – who is it? Explain your answer.} & \\
\end{align*}
\]

\[x \text{ and } y \text{ are both positive whole numbers.}

\[
\frac{x}{y} = 4
\]

\[x \text{ will always be a multiple of 4}

\[y \text{ will always be a factor of 4}

\text{Possible answer:}

\text{Dora is correct as } x \text{ will always have to divide into 4 equal parts e.g.} \\
\text{32 ÷ 8 = 4, 16 ÷ 4 = 4}

\text{Jack is incorrect.} \\
\text{40 ÷ 10 = 4 and 10 is not a factor of 4}

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Find Pairs of Values (2)

Notes and Guidance

Building on from the last step, children find possible solutions to equations which involve multiples of one or more unknown.

They should be encouraged to try one number for one of the variables first and then work out the corresponding value of the other variable. Children should then work systematically to test if there are other possible solutions that meet the given conditions.

Mathematical Talk

What does $2a$ mean? (2 multiplied by an unknown number)
What is the greatest/smallest number ‘$a$’ can be?

What strategy did you use to find the value of ‘$b$’?

Can you draw a bar model to represent the following equations:

$3f + g = 20$
$7a + 3b = 40$

What could the letters represent?

Varied Fluency

In this equation, $a$ and $b$ are both whole numbers which are less than 12.

$2a = b$

Write the calculations that would show all the possible values for $a$ and $b$.

Chose values of $x$ and use the equation to work out the values of $y$.

$7x + 4 = y$

<table>
<thead>
<tr>
<th>Value of $x$</th>
<th>Value of $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>

$2g + w = 15$
$g$ and $w$ are positive whole numbers.
Write down all the possible values for $g$ and $w$, show each of them in a bar model.

<table>
<thead>
<tr>
<th>$g$</th>
<th>$g$</th>
<th>$w$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>
Find Pairs of Values (2)

Reasoning and Problem Solving

\[ ab + b = 18 \]

Mo says,

Is Mo correct? Explain your answer.

Possible answer:

Mo is incorrect. Children may give examples to prove Mo is correct e.g. if \( a = 5 \) and \( b = 3 \), but there are also examples to show he is incorrect e.g. \( a = 2 \) and \( b = 6 \) where \( a \) and \( b \) are both even.

Large beads cost 5p and small beads cost 4p

Rosie has 79p to spend on beads.

How many different combinations of small and large beads can Rosie buy?

Can you write expressions that show all the solutions?

Possible answers:

- \( 3l + 16s \)
- \( 7l + 11s \)
- \( 11l + 6s \)
- \( 15l + s \)